Simulation of Ultrasound Systems using Field II

Summer School on Advanced Ultrasound Imaging 2023

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Purpose of lecture

Understanding medical ultrasound acoustic simulation and the signal processing in medical ultrasound systems. Give a hands-on knowledge of Field II by making an exercise using Matlab and the program.

The participant should have a portable PC, which has Matlab on it, with the latest version of Field II.

Purpose of Field II simulation

Simulation of medical ultrasound imaging to gain a detailed understanding of the acoustics and its influence on the signal processing, with the purpose of aiding the development of new advanced ultrasound systems and to reveal their realistic performance.

Lecture Outline

1. Simulation model: spatial impulse responses:
   - Linear description of acoustic fields using spatial impulse responses
   - Calculation of spatial impulse responses
   - Examples, problems and solutions: Time integration for improved accuracy
   - The Field II program
2. Simple uses of Field II for arrays
   - Calculation of emitted fields, CW, PSFs,
   - Calculation of intensities
   - How to calibrate the program
   - Attenuating medium

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Part 1: Spatial impulse responses

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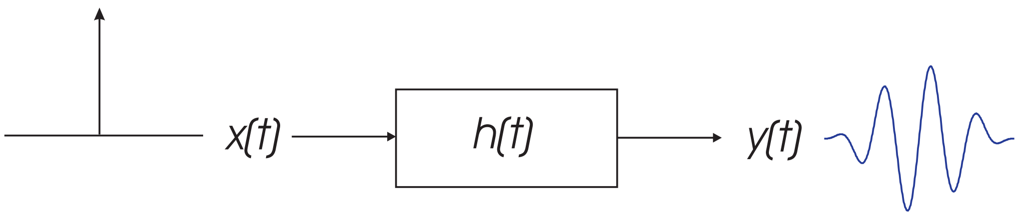
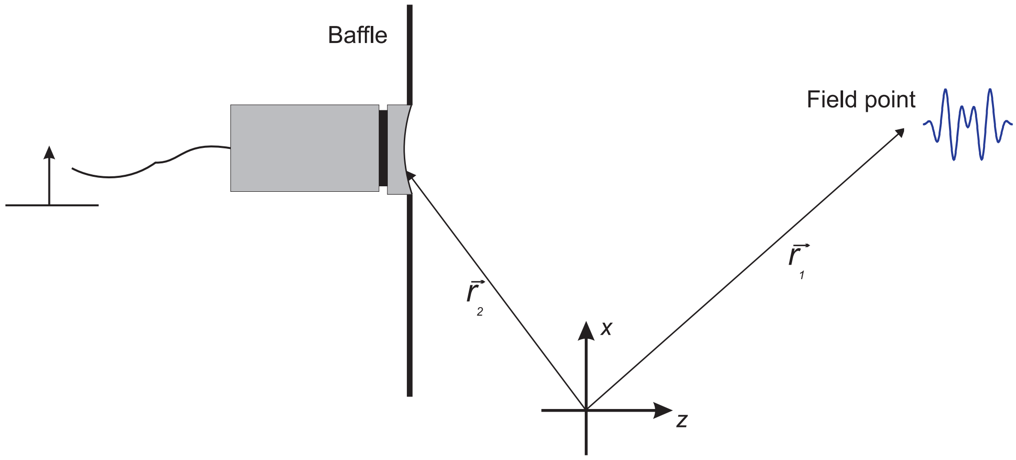
Linear Electrical System



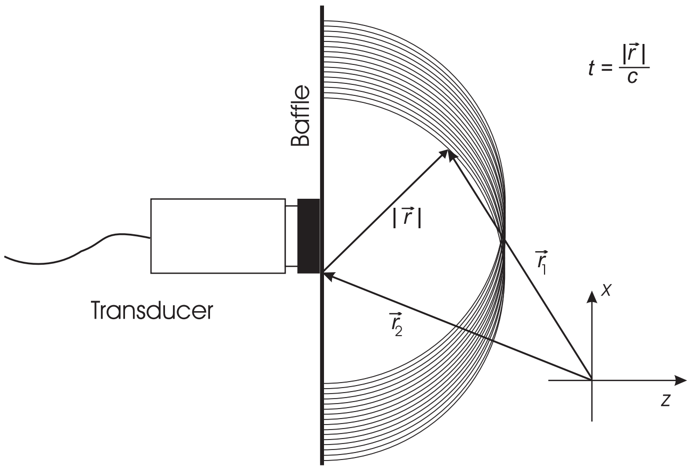
Fully characterized by it's impulse response

Linear Acoustic System



Impulse response at a point in space.

Huygens' Principle



Arrival times: $t = |\vec{r}|/c$
Moving the point results in a new impulse response:
Spatial Impulse Responses - $h(\vec{r}_1, t)$

Rayleigh's Integral

Summation of spherical waves from each point on the aperture surface:

$$p(\vec{r}_1,t) = \frac{\rho_0}{2\pi}\int_S \frac{\frac{\partial v_n(\vec{r}_2,t-\frac{|\vec{r}_1-\vec{r}_2|}{c})}{\partial t}}{|\vec{r}_1-\vec{r}_2|} d^2\vec{r}_2$$

$\mid \vec{r}_1 - \vec{r}_2 \mid$	- Distance to field point
$v_n(\vec{r}_2,t)$	- Normal velocity of transducer surface
ρ_0	- Density of medium

Derivation

Exchanging the integration and the partial derivative gives

$$p(\vec{r}_1,t) = \frac{\rho_0}{2\pi}\frac{\partial \int_S \frac{v_n(\vec{r}_2,t-\frac{|\vec{r}_1-\vec{r}_2|}{c})}{|\vec{r}_1-\vec{r}_2|}dS}{\partial t}.$$

Introduce the velocity potential ψ:

$$\vec{u}(\vec{r},t) = -\nabla\psi(\vec{r},t)$$

$$p(\vec{r},t) = \rho_0\frac{\partial\psi(\vec{r},t)}{\partial t}.$$

Only a scalar quantity need be calculated:

$$\psi(\vec{r}_1,t) = \int_S \frac{v_n(\vec{r}_2,t-\frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi\,|\vec{r}_1-\vec{r}_2|}dS$$

Derivation, continued .....

$$\psi(\vec{r}_1,t) = \int_S \frac{v_n(\vec{r}_2, t - \frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS$$

Excitation pulse can be separated from transducer geometry by introducing a time convolution with a δ-function:

$$\psi(\vec{r}_1,t) = \int_S \int_T \frac{v_n(\vec{r}_2,t_2)\delta(t - t_2 - \frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dt_2 dS,$$

Assume surface velocity is uniform over aperture making it independent of $\vec{r}_2$:

$$\psi(\vec{r}_1,t) = v_n(t) * \int_S \frac{\delta(t - \frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS,$$

$*$ denotes convolution.

Spatial impulse response

Summation of all spherical waves from the aperture:

$$h(\vec{r}_1,t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS$$

$\mid \vec{r}_1 - \vec{r}_2 \mid$	- Distance to field point
c	- Speed of sound
S	- Transducer surface

Ultrasound fields

Emitted field:

$$p(\vec{r}_1,t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1,t)$$

Pulse echo field:

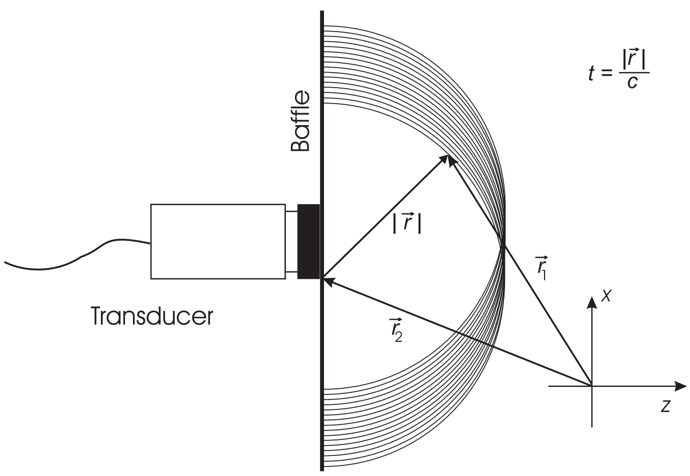
$$\begin{aligned} v_r(\vec{r}_1,t) &= v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1,t) \\ f_m(\vec{r}_1) &= \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c} \end{aligned}$$

Continuous wave fields:

$$\mathcal{F}\{p(\vec{r}_1,t)\}, \qquad \mathcal{F}\{v_r(\vec{r}_1,t)\}$$

All fields can be derived from the spatial impulse response.

Huygens' principle



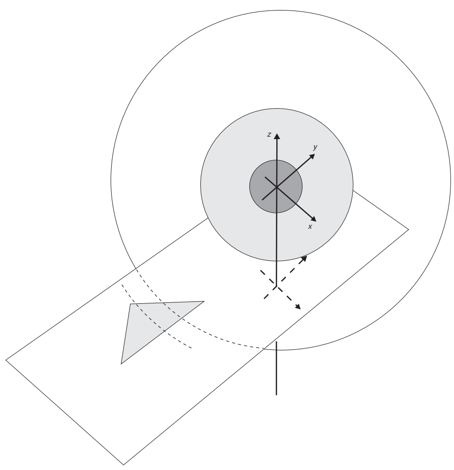
Arrival times: $t = |\vec{r}|/c$
Moving the point results in a new impulse response:
Spatial Impulse Responses - $h(\vec{r}_1,t)$

Acoustic Reciprocity

Kinsler & Frey:
”If in an unchanging environment the locations of a small source and a small receiver are interchanged, the received signal will remain the same.”

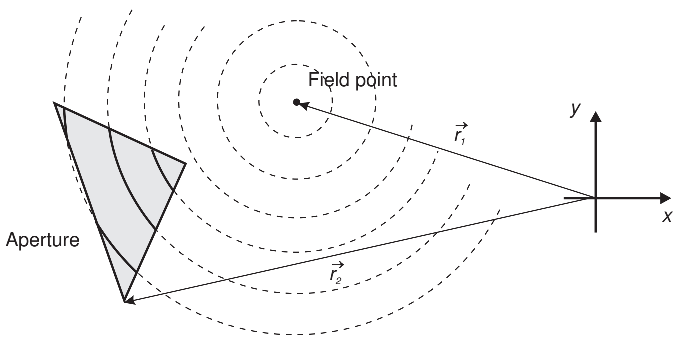
In other words:
The field can be derived by emitting a spherical wave from the field point and finding the arc that intersects the aperture.

Situation



Emission of spherical wave from the field point and its intersection of the aperture.

Projection onto Aperture Plane



Intersection of spherical waves from the field point by the aperture, when the field point is projected onto the plane of the aperture.

Calculation of Spatial Impulse Responses

Spatial impulse response:

$$h(\vec{r}_1,t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi|\vec{r}_1-\vec{r}_2|} dS,$$

$\vec{r}_1$ position of field point, $\vec{r}_2$ position on aperture.

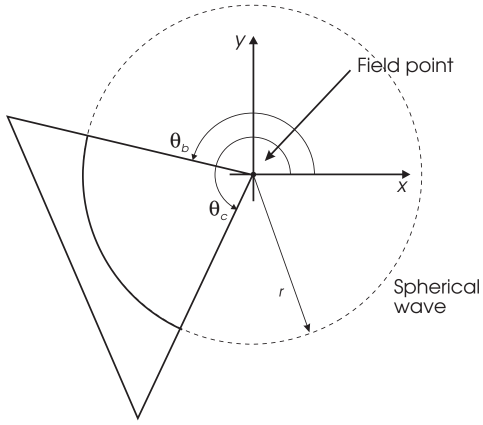
Polar coordinate system gives

$$\int\int_s f(x,y)dxdy = \int_0^r \int_0^{2\pi} r f(r,\theta) d\theta dr.$$

Projected circles have radius: $r = \sqrt{(ct)^2 - z^2}$

Distance to field point: $R = \sqrt{z^2 + r^2}$,
z - field point's height above $x - y$ plane.

Example



First response arrives at $t = t_1 = z/c$, hereafter the fixed part of the circle between the angles θ_b and θ_c contributes to the response.

Derivation

$$h_T(\vec{r}_1,t) = \int_0^r \int_{\theta_b}^{\theta_c} r\frac{\delta(t-\frac{|R|}{c})}{2\pi|R|}d\theta dr = \frac{\theta_c-\theta_b}{2\pi}\int_0^r r\frac{\delta(t-\frac{|R|}{c})}{|R|}dr$$

Substitution: $R = \sqrt{z^2+r^2}, dR/dr = \frac{1}{2}(z^2+r^2)^{-1/2}2r = \frac{1}{2R}2r$ leading to $2RdR = 2rdr$. This results in

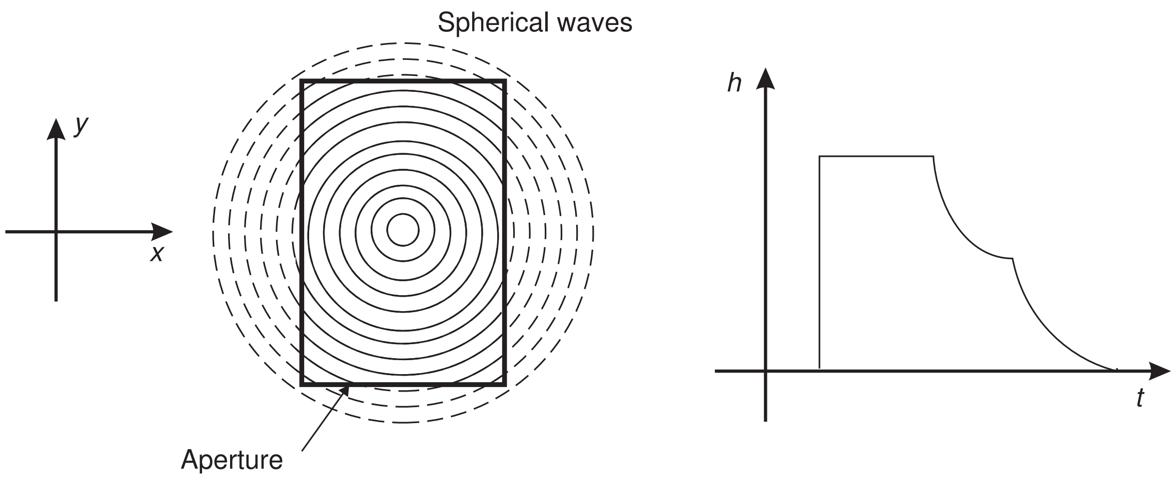
$$h_T(\vec{r}_1,t) = \frac{\theta_c-\theta_b}{2\pi}\int_z^{\sqrt{z^2+r^2}} R\frac{\delta(t-\frac{|R|}{c})}{|R|}dR = \frac{\theta_c-\theta_b}{2\pi}\int_z^{\sqrt{z^2+r^2}} \delta(t-\frac{|R|}{c})dR$$

Time substitution $R/c = t'$ results in

$$h_T(\vec{r}_1,t) = \frac{\theta_c-\theta_b}{2\pi}c\int_{t_1}^{t_x}\delta(t-t')dt' = \frac{(\theta_c-\theta_b)}{2\pi}c \qquad \text{for } t_1 \le t \le t_x$$

Time t_x equals the corresponding time for edge point closest to origo.

Examples of Spatial Impulse Responses

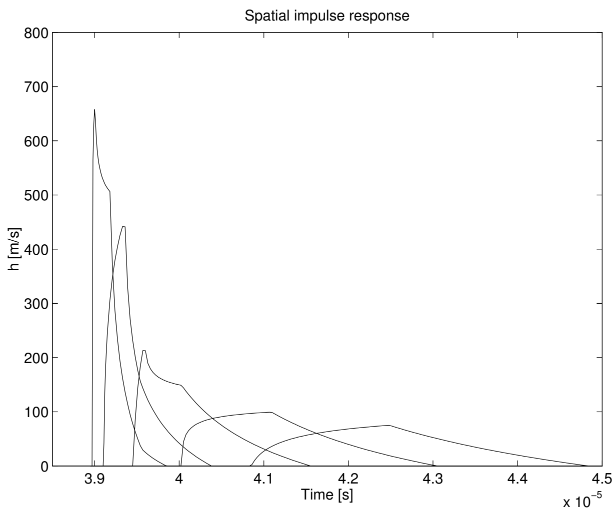


Emitted pressure field:

$$p(\vec{r},t) = \rho_0 \frac{\partial v_n(t)}{\partial t} * h(\vec{r},t)$$

Computer simulation: sir_demo.m

Triangular aperture



Spatial impulse responses calculated for a triangle at different positions.

Ultrasound fields

Emitted field:

$$p(\vec{r}_1,t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1,t)$$

Pulse echo field:

$$\begin{aligned} v_r(\vec{r}_1,t) &= v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1,t) \\ f_m(\vec{r}_1) &= \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c} \end{aligned}$$

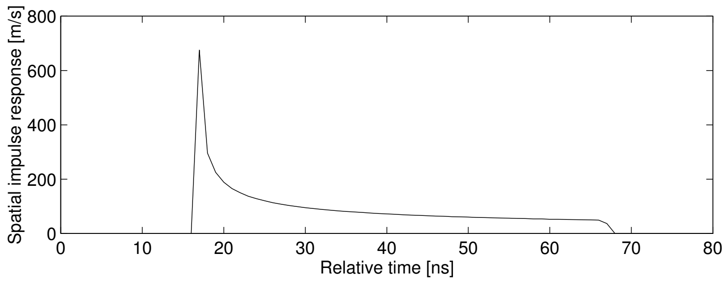
Continuous wave fields:

$$\mathcal{F}\left\{p(\vec{r}_1,t)\right\}, \qquad \mathcal{F}\left\{v_r(\vec{r}_1,t)\right\}$$

All fields can be derived from the spatial impulse response.

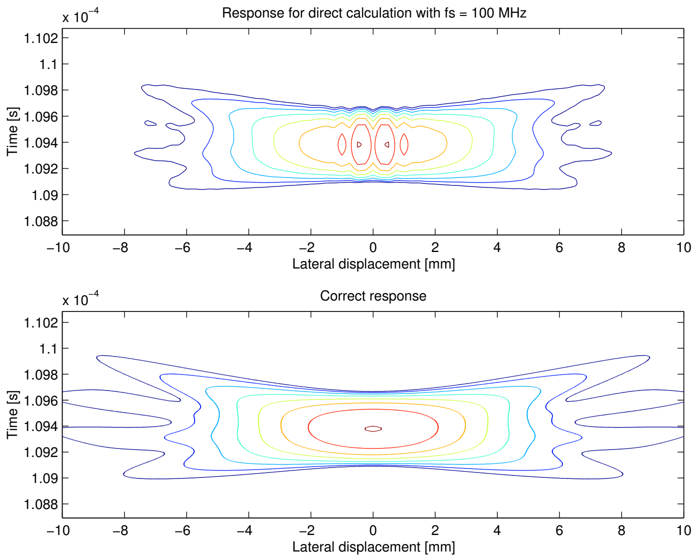
Problems with Spatial Impulse Responses

1. Not easy to calculate analytically for complex geometries and apodization over the aperture
2. Numerical difficulties
   - Edges (difficult in sampled system)
   - Very short responses (loss of energy)



Response from small array element

Problems with calculation of Spatial Impulse Responses



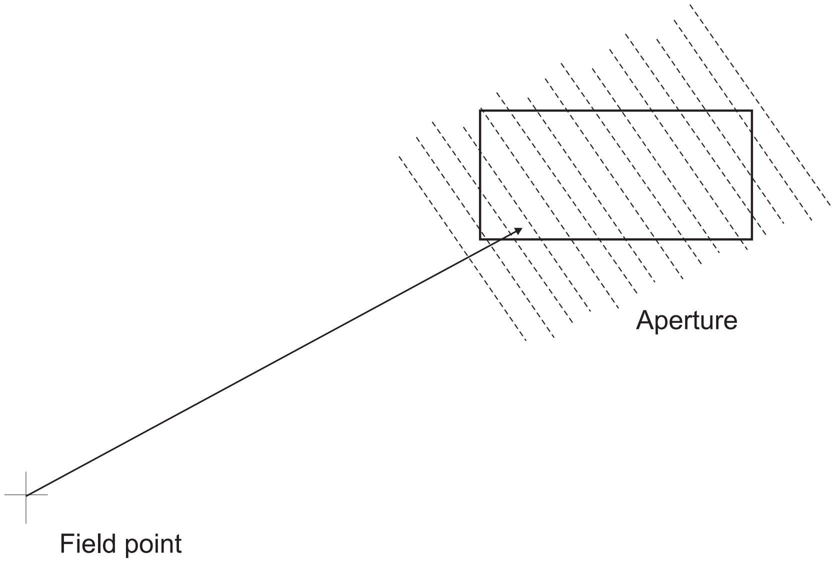
Pulse-echo field from concave transducer at focal point

Solutions

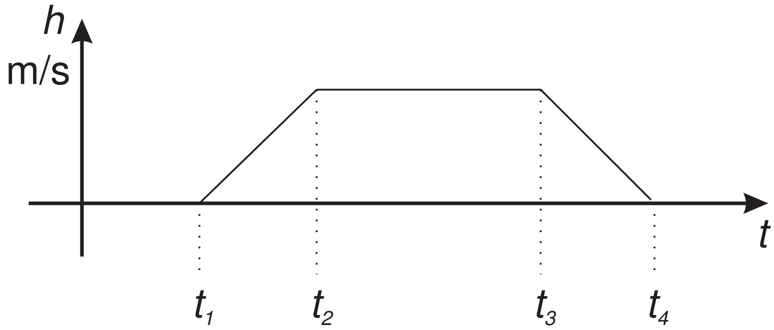
1. Linearity → superposition can be used
   → division into elementary elements (rectangles, triangles or bounding lines)

2. Limited bandwidth of pulse → energy important, not actual shape of response → time integration of spatial impulse response

Far-Field Response from Rectangular Element



Far-Field Response from Rectangular Element



Energy conservation: Integration of response over time.

Size of element (far-field response):

$$z \gg \frac{w^2}{4\lambda} \qquad w \ll \sqrt{4z\lambda}$$

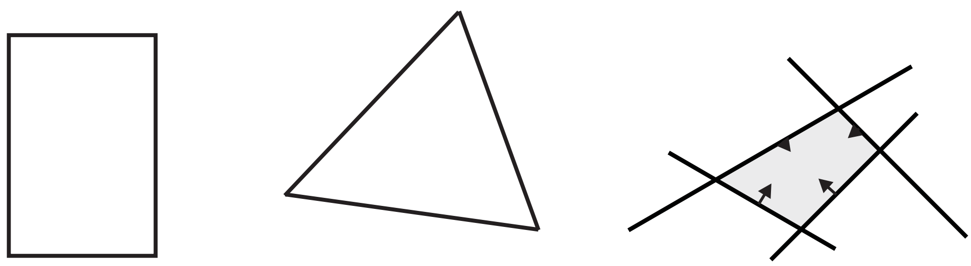
z - distance to field point, w - largest dimension of rectangle, λ - wavelength

Modeling

Possibilities:

1. All transducer geometries
2. Phasing
3. Apodization ($v(t)$ varies over the surface)
4. All kinds of excitations

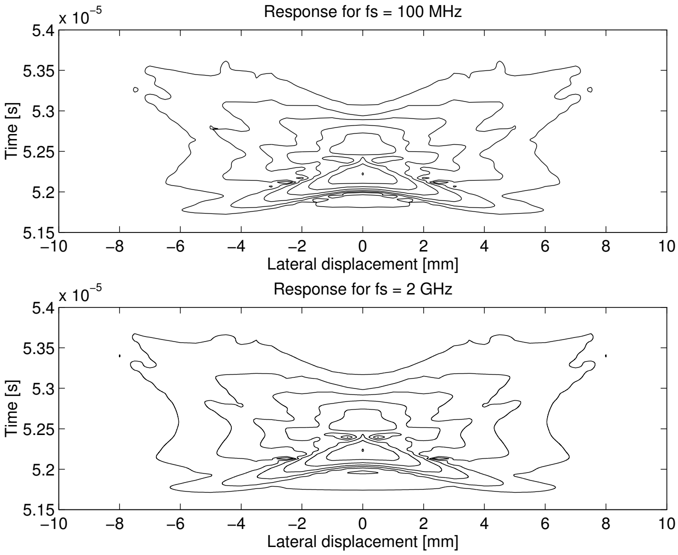
Basic geometries



Method:

- Rectangles: Direct integration of far field response
- Triangles: Romberg integration
- Bounding lines: Romberg integration

Point spread functions (rectangles)

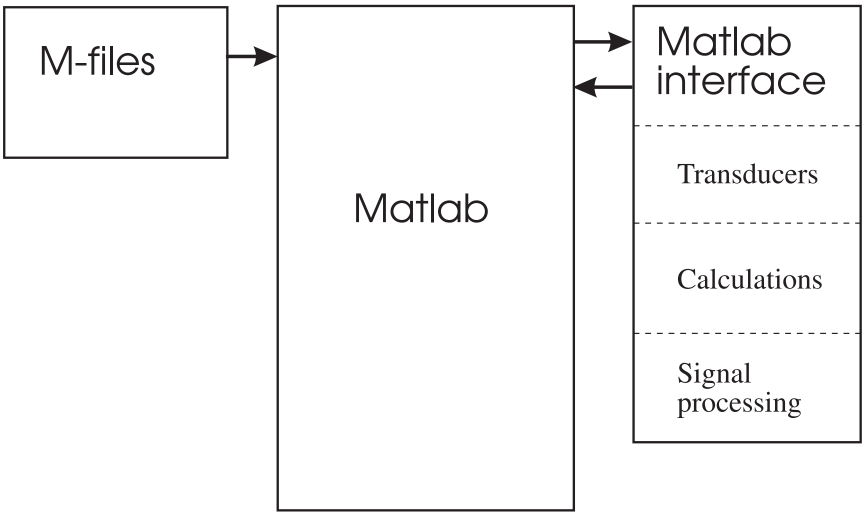


Point spread function for 64 element linear array for $f_s = 100$ MHz (top) and $f_s = 2$ GHz (bottom). (6 dB between the contour lines)

Field II

- Transducer modeled by dividing it into rectangles, triangles or bounding lines.
- C program interfaced to Matlab.
- Matlab used as front-end.
- Can handle any transducer geometry.
- Physical understanding of transducer.
- Pre-defined types: piston and concave single element, linear array, phased array, convex array, 2D matrix
- Any focusing, apodization, and excitation pulse.
- Multiple focusing and apodization.
- Dynamic focusing.
- Can calculate all types of fields (emitted, received, pulsed, CW)
- Can generate artificial ultrasound images (phased and linear array images with multiple receive and transmit foci).
- Data storage not necessary.
- Post-processing in Matlab
- Versions for: Windows, Linux, Apple OS-X
- Free program at: http://field-ii.dk/

Field II Program Organization



Makes it possible to use Matlab for signal processing and imaging

Using the Field II program 1 (field demo.m)

```
%  Start the system and initialize the path

path(path,'/home/jaj/programs/field_II/M_files')
path(path,'/home/jaj/programs/field_II/m_utilities');

%  Initialize the field system

field_init

%  Set basic parameters

f0=1e6;                  %  Transducer center frequency [Hz]
fs=100e6;                %  Sampling frequency [Hz]
c=1540;                  %  Speed of sound [m/s]
density=1e6;             %  Density [g/m^3]
lambda=c/f0;             %  Wavelength [m]
radius=10/1000;          %  Radius of piston transducer [m]
```

Using the Field II program 2 -Aperture definition

```
%  Generate an aperture

dist_field=1/1000;
ele_size=sqrt(dist_field*4*lambda);
aperture = xdc_piston (radius, ele_size);
xdc_show(aperture)
show_xdc(aperture)
ele_size=0.1/1000;
aperture = xdc_piston (radius, ele_size);

%  Set the impulse response and excitation of the aperture

impulse_response=sin(2*pi*f0*(0:1/fs:2/f0));
impulse_response=impulse_response.*hanning(max(size(impulse_response)))';
xdc_impulse (aperture, impulse_response);

excitation=sin(2*pi*f0*(0:1/fs:2/f0));
excitation=excitation.*hanning(max(size(excitation)))';
xdc_excitation (aperture, excitation);
```


Using the Field II program 3 - Field calculation

```
%  Make a calculation of the spatial impulse response

[h,t] = calc_h (aperture, [0 0 10]/1000);
plot((0:length(h)-1)/fs+t, h); xlabel('Time [s]'); ylabel('h [m/s]')
[h,t] = calc_h (aperture, [2 0 10]/1000);
plot((0:length(h)-1)/fs+t, h); xlabel('Time [s]'); ylabel('h [m/s]')
[h,t] = calc_h (aperture, [5 0 10]/1000);
plot((0:length(h)-1)/fs+t, h); xlabel('Time [s]'); ylabel('h [m/s]')
[h,t] = calc_h (aperture, [8 0 10]/1000);
plot((0:length(h)-1)/fs+t, h); xlabel('Time [s]'); ylabel('h [m/s]')
[h,t] = calc_h (aperture, [20 0 10]/1000);
plot((0:length(h)-1)/fs+t, h); xlabel('Time [s]'); ylabel('h [m/s]')

%  Make calculations for a number of points

points=[0:10; zeros(1,11); 10*ones(1,11)]'/1000
[h,t] = calc_h (aperture, points);
plot((0:length(h)-1)/fs+t, h) xlabel('Time [s]'); ylabel('h [m/s]')
```

Using the Field II program 4 - Emitted field

```
%  Make a calculation of the emitted pressure

[p,t] = calc_hp (aperture, [0 0 50]/1000);
plot((0:length(p)-1)/fs+t, p*density); xlabel('Time [s]'); ylabel('p [Pa]')
points=[0:10; zeros(1,11); 50*ones(1,11)]'/1000
[p,t] = calc_hp (aperture, points);
plot((0:length(p)-1)/fs+t, p*density); xlabel('Time [s]'); ylabel('p [Pa]')

%  Make a calculation of the pulse-echo voltage

[v,t] = calc_hhp (aperture, aperture, [0 0 50]/1000);
plot((0:length(v)-1)/fs+t, v)
```


Using the Field II program 5 - Setting parameters

```
%  Setting parameters

fs=1000e6;                %  Sampling frequency [Hz]
set_field ('fs',fs);

%  Set the impulse response and excitation of the aperture again

impulse_response=sin(2*pi*f0*(0:1/fs:2/f0));
impulse_response=impulse_response.*hanning(max(size(impulse_response)))';
xdc_impulse (aperture, impulse_response);

excitation=sin(2*pi*f0*(0:1/fs:2/f0));
excitation=excitation.*hanning(max(size(excitation)))';
xdc_excitation (aperture, excitation);

[h,t] = calc_h (aperture, [8 0 10]/1000);
plot((0:length(h)-1)/fs+t, h)

%  Release the apertures
xdc_free (aperture);

%  Shut down field
field_end
```

Simulation of Ultrasound Systems using Field II

Part 2: Imaging with arrays

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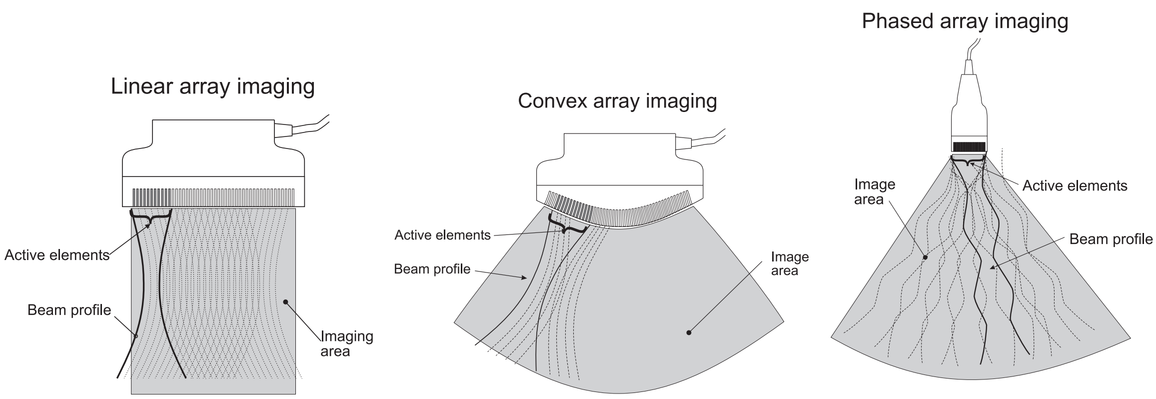
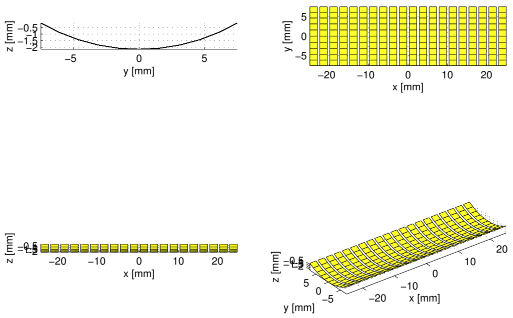
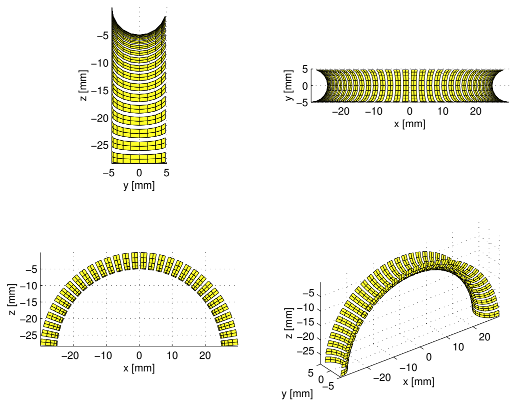
Conventional imaging methods



Typical transducers

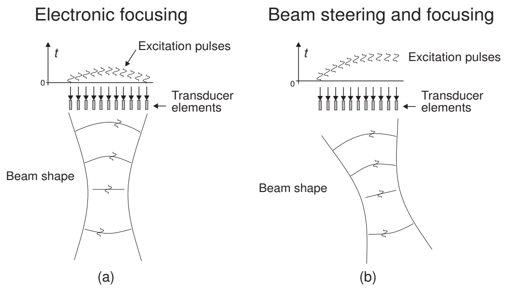


Linear or phased array



Convex array

Focusing and beamforming



Time from the center of aperture to field point:

$$t_i = \frac{1}{c}\sqrt{(x_i - x_f)^2 + (y_i - y_f)^2 + (z_i - z_f)^2}$$

(x_f, y_f, z_f) - position of the focal point (x_i, y_i, z_i) - center of physical element number i,
Reference point on aperture:

$$t_c = \frac{1}{c}\sqrt{(x_c - x_f)^2 + (y_c - y_f)^2 + (z_c - z_f)^2}$$

(x_c, y_c, z_c) - reference center point on the aperture.
Delay to use on each element of the array:

$$\Delta t_i = \frac{1}{c}\left(\sqrt{(x_c - x_f)^2 + (y_c - y_f)^2 + (z_c - z_f)^2} - \sqrt{(x_i - x_f)^2 + (y_i - y_f)^2 + (z_i - z_f)^2}\right)$$

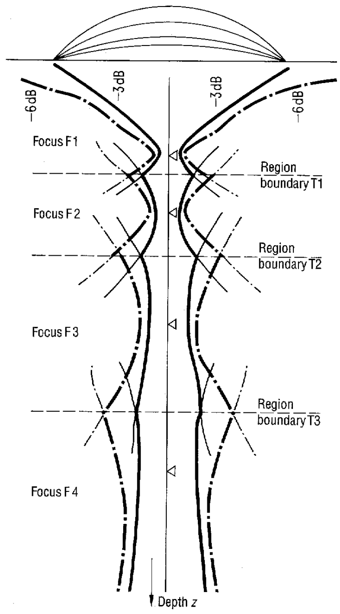
Field II: Focusing and apodization time lines

Focusing:

From time	Focus at
0	x_1, y_1, z_1
t_1	x_1, y_1, z_1
t_2	x_2, y_2, z_2
$\vdots$	$\vdots$

Apodization:

From time	Apodize with
0	$a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$
t_1	$a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$
t_2	$a_{2,1}, a_{2,2}, \cdots a_{2,N_e}$
t_3	$a_{3,1}, a_{3,2}, \cdots a_{3,N_e}$
$\vdots$	$\vdots$



Program example - Transducer and phantom definition

```
%  Generate aperture for emission and set impulse response

emit_aperture = xdc_linear_array (N_elements, width, element_height, kerf,
                                  1, 1,focus);
xdc_impulse (emit_aperture, impulse_response);
xdc_excitation (emit_aperture, excitation);

%  Generate aperture for reception

receive_aperture = xdc_linear_array (N_elements, width, element_height, kerf,
                                     1, 1,focus);
xdc_impulse (receive_aperture, impulse_response);

%   Load the computer phantom

[phantom_positions,phantom_amplitudes]= cyst_phantom(50000);
```

Program example - Simulation of linear array imaging

```
%  Perform the image simulation
x= -image_width/2;
for i=1:no_lines
   %   Set the focus and apodization for this direction

  xdc_center_focus (emit_aperture, [x 0 0]);
  xdc_focus (emit_aperture, t0, [x 0 z_focus]);
  xdc_center_focus (receive_aperture, [x 0 0]);
  xdc_focus (receive_aperture, focus_times, [x*ones(Nf,1), zeros(Nf,1), focal_zones]);
  xdc_apodization (emit_aperture,    t0, apo_vector);
  xdc_apodization (receive_aperture, t0, apo_vector);

  %   Calculate the received response
  [v, t1]=calc_scat(emit_aperture, receive_aperture,
                    phantom_positions, phantom_amplitudes);
  %  Store the result
  image_data(1:max(size(v)),i)=v;
  times(i) = t1;

  %  Move the beam
  x = x + d_x;
  end
```


Program example - Simulation of phased array imaging

```
      %  Initialize is the same as before

angles=90;                               %  Degrees
no_lines=100;                            %  Number of lines in image
emit_r=40/1000;                          %  Emission focus[m]
focal_zones=[10:5:100]/1000;             %  Receive focal zones [m]
focus_times=(focal_zones-2.5/1000)/c;    %  Receive focal times [s]

     %  Do the imaging

dtheta=angles/no_lines/180*pi;
theta= -angles/2/180*pi;
for i=[1:no_lines]

        %   Set the focus for this direction

  xdc_center_focus (emit_aperture, [0 0 0]);
  xdc_focus (emit_aperture, t0, [emit_r*sin(theta) 0 emit_r*cos(theta)]);
  xdc_center_focus (receive_aperture, [0 0 0]);
  xdc_focus (receive_aperture, focus_times, [focal_zones*sin(theta) 0 ...
                                             focal_zones*cos(theta)]);
```

Program example - Phased array imaging cont.

```
      %   Calculate the received response

  [rf_data, tstart]=calc_scat(emit_aperture, receive_aperture,
                              phantom_positions, phantom_amplitudes);

      %  Store the result

  data(1:length(rf_data),i)=rf_data;
  start_times(i)=tstart;

      %  Move the beam

  theta=theta+dtheta;
  end

    %  Make the image

make_image
```


Calibration for emitted field

Calculated by Field II:

$$p(\vec{r}_1,t) = e(t) * v_t(t) * h(\vec{r}_1,t)$$

$e(t)$ - Excitation voltage applied onto transducer

$v_t(t)$ - Impulse response from voltage to front face acceleration

Both initially set to δ-functions

Emitted field:

$$p(\vec{r}_1,t) = \rho_0 \frac{\partial v(t)}{\partial t} * e(t) * h(\vec{r}_1,t)$$

Calibration: $v_t(t) = \rho_0 \frac{\partial v(t)}{\partial t}$

Calibration Measurement

- Place hydrophone at focus or in the very far field, so that $h(\vec{r}_1,t) \approx h_k \delta(t - |\vec{r}_1|/c)$
- Apply pseudo random noise to transducer and measure response
- Cross-correlation $R_{12}(\tau)$ between excitation and measured response gives:

$$\begin{aligned}
R_{12}(\tau) &= E\{e(t)p(\vec{r}_1,t+\tau)\} = E\{e(t)\rho_0 \frac{\partial v(t)}{\partial t} * e(t+\tau) * h(|\vec{r}_1|,t)\} \\
&= E\{e(t)e(t+\tau) * \rho_0 \frac{\partial v(t)}{\partial t} * k_h \delta(t - |\vec{r}_1|/c)\} \\
&= R_e(\tau) * \rho_0 k_h \frac{\partial v(\tau - |\vec{r}_1|/c)}{\partial \tau} = \sigma_0^2 k_h \rho_0 \frac{\partial v(\tau - |\vec{r}_1|/c)}{\partial \tau} \\
&= \sigma_0^2 k_h v_t(\tau - |\vec{r}_1|/c)
\end{aligned}$$

$e(t)$ - White, random signal, Power: σ_0^2

$h(\vec{r}_1,t) \leftrightarrow H(\vec{r}_1,f)$ calibration constant: $k_h = H(\vec{r}_1,f_0)$, f_0 - Transducer center frequency

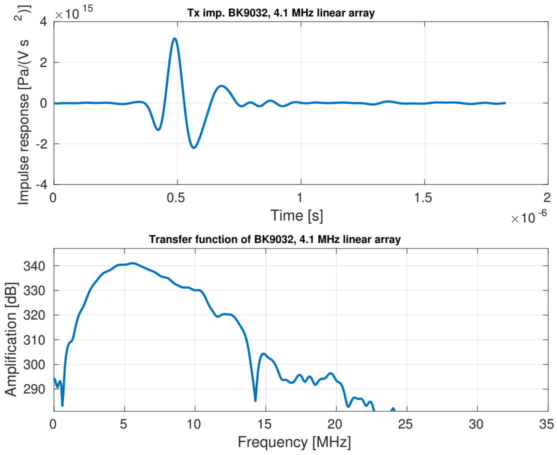
Calibration Measurement II

Scaled impulse response to use in Field II $v_t(k)$ is then:

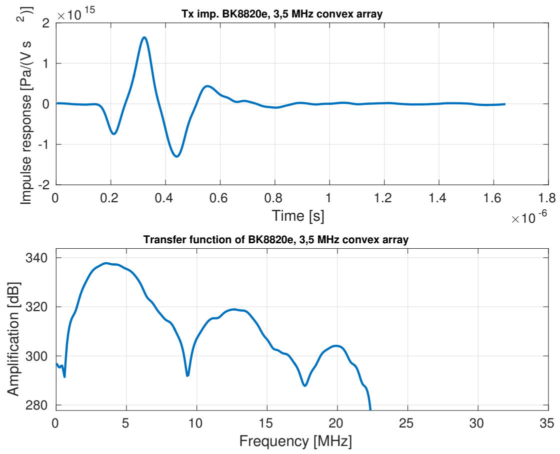
$$v_t(k) = \frac{R_{12}(k) f_s^2}{\sigma_0^2 k_h}, \quad (1)$$

as the convolution operation in Field II includes a division with the sampling frequency for each convolution to yield results independent of a change in sampling frequency.

Measured impulse responses I

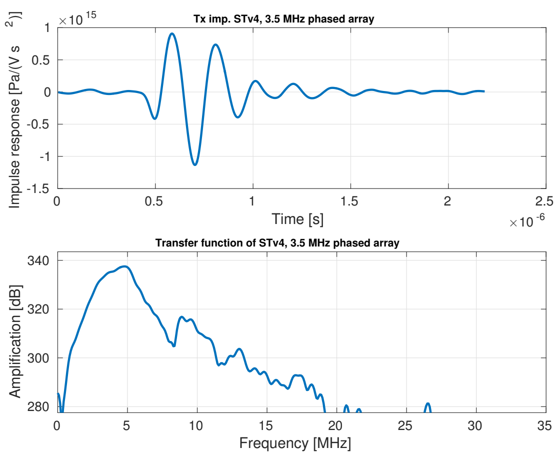


Linear array

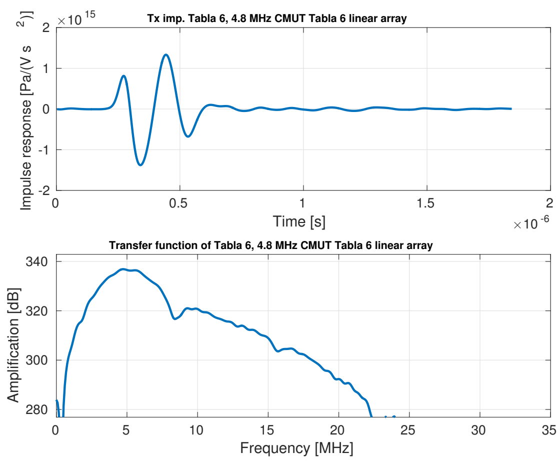


Convex array

Measured impulse responses II



Phased array



Linear CMUT array

Values for impulse response and excitation

Excitation:

$$e(t) = \sin(2\pi f_0 t), \quad 0 \le t \le M/f_0$$

M: 1 - 2 periods for B-mode imaging or 4-8 periods for flow imaging.

Impulse response:

$$v_t(t) = \sin(2\pi f_0 t) \cdot hanning(t), \quad 0 \le t \le M/f_0$$

M: 1 - 2 periods for broad band transducers

More realistic minimum phase impulse responses can be designed using the **buttord** and **butter** commands in Matlab.

Intensity Calculation and Calibration

Spatial peak temporal average intensity:

$$I_{spta} = \frac{1}{T_{prf}} \int_0^{T_{prf}} \frac{p^2(\vec{r}_1, t)}{\rho c} dt$$

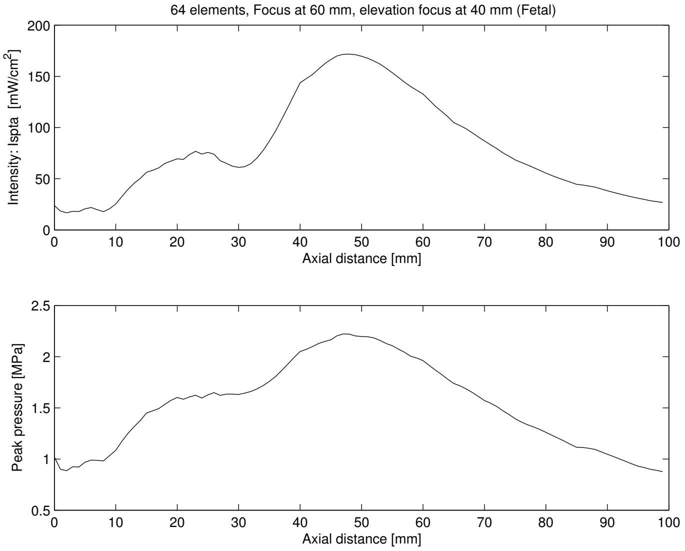
T_{prf} - Time between pulse emissions

ρc - Characteristic acoustic impedance

Method:

- Calculate intensity profile
- Scale excitation voltage to meet correct peak intensity

Intensity example



Attenuating medium

Attenuated spatial impulse response:

$$h_{att}(t,\vec{r}) = \int_T \int_S a(t-\tau,|\vec{r}|)\frac{\delta(\tau-\frac{|\vec{r}|}{c})}{|\vec{r}|}dSd\tau$$

Amplitude of attenuation transfer function:

$$|A(f,|\vec{r}|)| = \exp(-\alpha|\vec{r}|)\exp(-\beta(f-f_0)|\vec{r}|)$$

Assuming a minimum phase attenuation results in:

$$\begin{aligned} A(f,|\vec{r}|) &= \exp(-\alpha|\vec{r}|)\exp(-\beta(f-f_0)|\vec{r}|) \times \exp(-j2\pi f(\tau_b + \tau_m\frac{\beta}{\pi^2}|\vec{r}|) \\ &\times \exp(j\frac{2f}{\pi}\beta|\vec{r}|\ln(2\pi f)) \end{aligned}$$

τ_b is bulk propagation delay per unit length and equals $1/c$.

τ_m is minimum phase delay factor. Gurumurthy and Arthur (1982) suggests a value of 20 to fit dispersion in tissue.

Parameters in Field II

Attenuation transfer function used in Field II:

$$A(f,|\vec{r}|) = \exp(-\alpha|\vec{r}|)\exp(-\beta(f-f_0)|\vec{r}|)\times\exp(-j2\pi f(\tau_b+\tau_m\frac{\beta}{\pi^2}|\vec{r}|)$$
$$\times \exp(j\frac{2f}{\pi}\beta|\vec{r}|\ln(2\pi f))$$

α - Frequency independent attenuation at the frequency f_0 [dB/m]
β - Frequency dependent attenuation factor around f_0 [dB/m Hz]
$\tau_m = 20$, $\tau_b = 1/c$
r - Distance from center of element to field point

Setting attenuation in Field II

α - Frequency independent attenuation at the frequency f_0 [dB/m]
β - Frequency dependent attenuation factor around f_0 [dB/m Hz]

Note that α and β should correspond, so that $\alpha = f_0\beta$.

So for 0.5 dB/[MHz cm] around $f_0 = 3$ MHz use this:

```
set_field ('att',1.5*100);
set_field ('Freq_att',0.5*100/1e6);
set_field ('att_f0',3e6);
set_field ('use_att',1);
```

Simulation of Ultrasound Systems using Field II

Part 3: Scattered fields and imaging

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Outline

- Derivation of wave equation
- Finding the scattered field
- Finding the received signal
- Simulating imaging using Field II
- Examples of use: Simulating in-vivo B-mode images

Notes: JAJ: Pages 21 - 57 + Field II Users' guide

Basic equations

Constitutive (pressure-density relations, material):

$$\frac{1}{c^2}\frac{\partial p_1}{\partial t} = \frac{\partial \rho_1}{\partial t} + \vec{u}\cdot\nabla\rho$$

Dynamic (description of motion):

$$\rho_{ins}\frac{d\vec{u}}{dt} = -\nabla P_{ins}$$

Continuity (conservation of mass):

$$\frac{\partial \rho_{ins}}{\partial t} = -\nabla\cdot(\rho_{ins}\vec{u})$$

Quantities:

P	- mean pressure of medium,	p_1	- pressure variation due to ultrasound wave
ρ	- density of undisturbed medium,	$\Delta\rho(\vec{r})$	- small density variation in tissue
ρ_1	- density change due to ultrasound wave		
c	- speed of sound,	$\Delta c(\vec{r})$	- small speed of sound variation in tissue

Wave equation

Approximations:

$$\begin{aligned} P_{ins}(\vec{r},t) &= P + p_1(\vec{r},t) \\ \rho_{ins}(\vec{r},t) &= \rho(\vec{r}) + \rho_1(\vec{r},t) \\ \rho(\vec{r}) &= \rho_0 + \Delta\rho(\vec{r}), \quad c(\vec{r}) = c_0 + \Delta c(\vec{r}) \end{aligned}$$

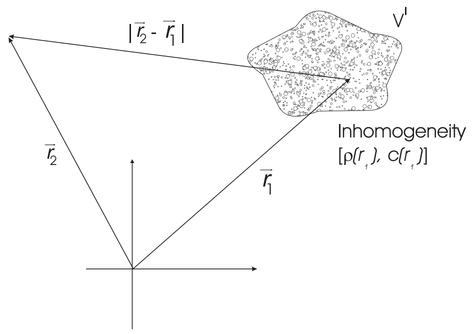
Mix basic equations and use small perturbation approximations gives the wave equation:

$$\nabla^2 p_1 - \frac{1}{c_0^2}\frac{\partial^2 p_1}{\partial t^2} = -\frac{2\Delta c}{c_0^3}\frac{\partial^2 p_1}{\partial t^2} + \frac{1}{\rho_0}\nabla(\Delta\rho)\cdot\nabla p_1$$

Left side: standard wave propagation

Right side: terms accounting for scattering

Scattered field



$$p_s(\vec{r}_2,t) = \int_{V'}\int_T \left[\frac{1}{\rho_0}\nabla(\Delta\rho(\vec{r}_1))\cdot\nabla p_1(\vec{r}_1,t_1) - \frac{2\Delta c(\vec{r}_1)}{c_0^3}\frac{\partial^2 p_1(\vec{r}_1,t_1)}{\partial t^2}\right] G(\vec{r}_1,t_1 \mid \vec{r}_2,t)dt_1\, d^3\vec{r}_1$$

G is Green's function:

$$G(\vec{r}_1,t_1 \mid \vec{r}_2,t) = \frac{\delta(t - t_1 - \frac{|\vec{r}_2-\vec{r}_1|}{c_0})}{4\pi \mid \vec{r}_2 - \vec{r}_1 \mid}$$

Born approximation

Define scattering operator:

$$F_{op} = \frac{1}{\rho_0}\nabla(\Delta\rho(\vec{r}_1))\cdot\nabla \qquad - \frac{2\Delta c(\vec{r}_1)}{c_0^3}\frac{\partial^2}{\partial t^2}$$

$$G_i = \text{integration operator (volume \& time)}$$

Pressure field inside scattering region is:

$$p_1(\vec{r},t) = p_i(\vec{r},t) + p_s(\vec{r},t)$$

p_i - incident pressure field
p_s - scattered pressure field

Born approximation:

$$\begin{aligned} p_{s_b}(\vec{r}_2,t) &= G_iF_{op}\left[p_i(\vec{r}_1,t_1) + G_iF_{op}\{p_i(\vec{r}_1,t_1) + ...\}\right] \\ &= G_iF_{op}\,p_i(\vec{r}_1,t_1) + [G_iF_{op}]^2 p_i(\vec{r}_1,t_1) + ... \end{aligned}$$

Keep only first term (first order Born approximation)

$$p_{s_1}(\vec{r}_2,t) = G_iF_{op}p_i(\vec{r}_1,t_1)$$

Final solution to wave equation:

Received voltage signal:

$$p_r(\vec{r}_5,t) = \frac{\rho_0}{2c_0^2}\frac{\partial^2 E_m(t)}{\partial t^2} \underset{t}{\star} \frac{\partial v(t)}{\partial t} \underset{t}{\star} \int_{V'} \left[\frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0}\right] h_{pe}(\vec{r}_1,\vec{r}_5,t)d^3\vec{r}_1$$
$$= v_{pe}(t) \underset{t}{\star} f_m(\vec{r}_1) \underset{r}{\star} h_{pe}(\vec{r}_1,t)$$

Electrical impulse response: $v_{pe}(t) = \frac{\rho_0}{2c_0^2}\frac{\partial^2 E_m(t)}{\partial t^2} \underset{t}{\star} \frac{\partial v(t)}{\partial t}$

Transducer spatial response: $h_{pe}(\vec{r}_1,\vec{r}_5,t) = h(\vec{r}_1,\vec{r}_5,t) \underset{t}{\star} h(\vec{r}_5,\vec{r}_1,t)$

Scattering term: $f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0}$

Ultrasound fields

Emitted field:

$$p(\vec{r}_1,t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1,t)$$

Pulse echo field:

$$v_r(\vec{r}_1,t) = v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1,t)$$

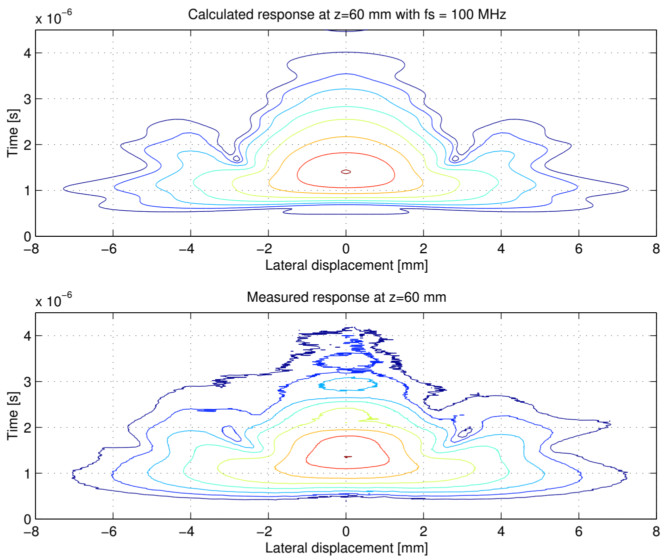
$$f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c}$$

Continuous wave fields:

$$\mathcal{F}\{p(\vec{r}_1,t)\}, \qquad \mathcal{F}\{v_r(\vec{r}_1,t)\}$$

All fields can be derived from the spatial impulse response.

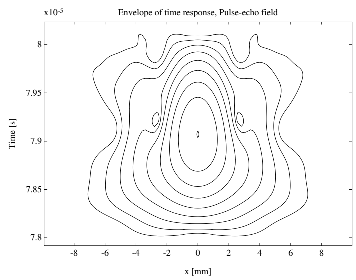
Point spread functions



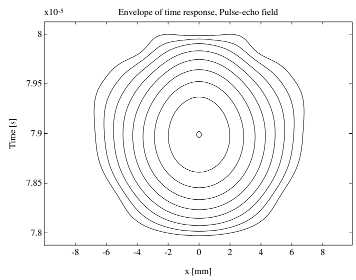
Point spread function for concave, focused transducer

top: simulation top   bottom: tank measurement   (6 dB contour lines)

Examples

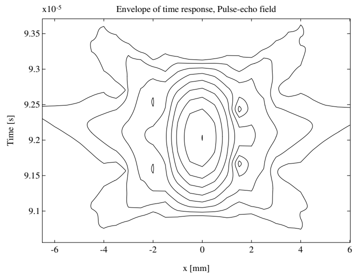


Concave transducer. 8 mm radius and focused at 100 mm. z = 60 mm.

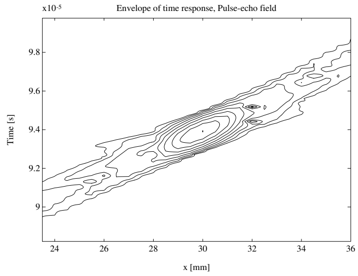


Same transducer with a Gaussian apodization.

Phased array



Response of 64 elements phased array focused at (0,0,70) mm.



Response of 64 elements phased array focused at (30,0,70) mm.

Calculation of Continuous wave field

Emitted pressure:

$$P(f) = \mathcal{F}\{p(\vec{r}_1, n)\} = \sum_{n=0}^{N} p(\vec{r}_1, n) \exp(-j2\pi f n \Delta T) \exp(-j2\pi f t_0)$$

N - Samples in impulse response

ΔT - Sampling interval

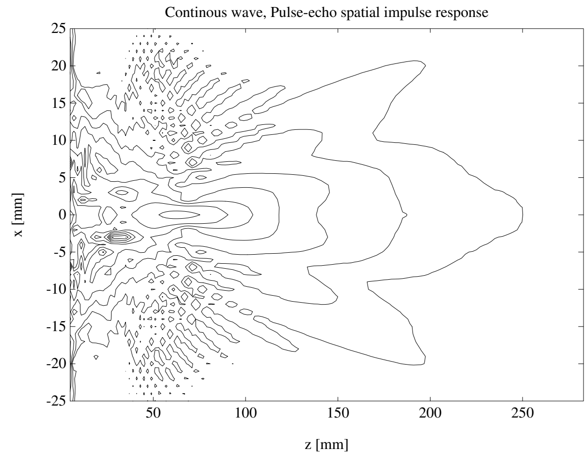
t_0 - Time for first sample in response

Pulse-echo field:

$$P(f) = \mathcal{F}\{v_r(\vec{r}_1, n)\} = \sum_{n=0}^{N} v_r(\vec{r}_1, n) \exp(-j2\pi f n \Delta T) \exp(-j2\pi f t_0)$$

Both can be found for any frequency.

Continuous wave field



Continuous wave field from a 32 $\times$ 32 elements 2D matrix transducer at 3 MHz. Different elements used in transmit and receive.

Realistic simulation of in-vivo imaging

Scattered field:

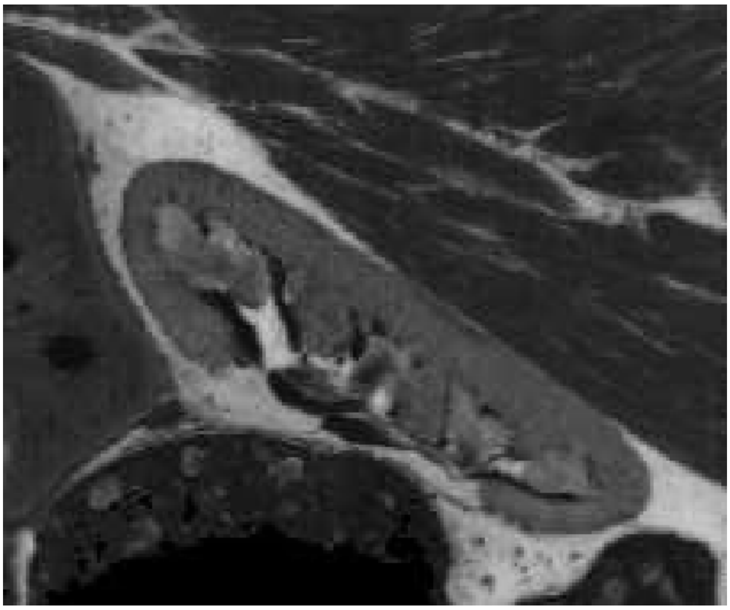
$$v_r(\vec{r}_1,t) = v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1,t)$$
$$f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c}$$

$\Delta\rho(\vec{r}_1)$ - Spatial variation in density
$\Delta c(\vec{r}_1)$ - Spatial variation in speed of sound

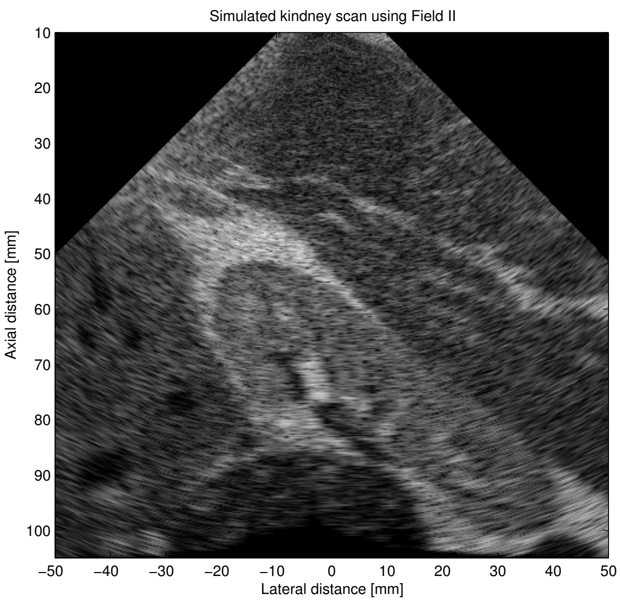
Description of spatial variation in backscattering from anatomic image:

$$\sigma_{f_m}(\vec{r}_1)$$

Scattering data from Visible Human Project

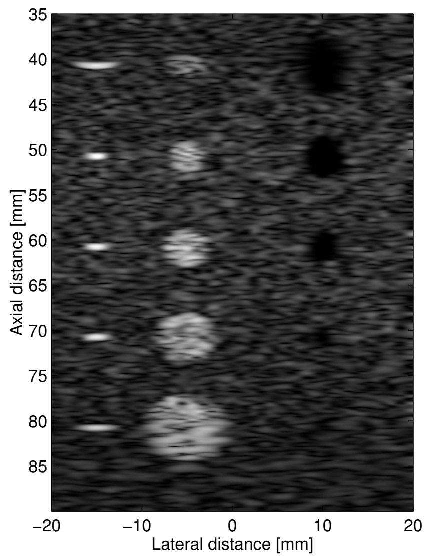


Simulation result for artificial kidney



Simulation can be done in parallel for multiple image lines

Cyst phantom



Conclusion

- You should by now have an understanding of both the theory and function of the Field II program and its features
- Any kind of transducer, excitation, impulse response, focusing and apodization can be simulated
- Simulations are done in C
- Scripting and pre- and post processing are done in Matlab
- All linear ultrasound imaging systems can be simulated including anatomic and flow systems
- Conventional and synthetic aperture systems can be simulated. Detailed in next lecture
- Simulations and measurements are accurate for both point spread functions, images, and flow modeling, which will described during the week.
- Simulations are easy to parallelize for shared disk, heterogeneous systems on multiple computers