Notes for the International Summer School on Advanced Ultrasound Imaging 2023

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CHAPTER

ONE

Introduction

These notes have been prepared for the international summer school on advanced ultrasound imaging in June 2023 at the Technical University of Denmark. The intended audience is Ph.D. students working in medical ultrasound. A knowledge of general linear acoustics and signal processing is assumed.

The notes give a linear description of general ultrasound imaging through the use of spatial impulse responses. It is shown in Chapter 2 how both the emitted and scattered fields for the pulsed and continuous wave case can be calculated using this approach. Chapter 3 gives a brief overview of modern ultrasound imaging and how it is simulated using spatial impulse responses. The first two chapters are based on the previous summer school course notes in [2]. Chapter 5 gives a brief description of both spectral and color flow imaging systems and their modeling and simulation along with the more modern vector velocity systems. This is based on the book chapter in [3]. A description of synthetic aperture imaging is given in Chapter 4 based on [4].

For the summer school it is assumed that the participant has read and understands the first two chapters on linear imaging. Lectures will be given on the content of the other chapters.

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CHAPTER

TWO

Description of ultrasound fields

This chapter gives a linear description of acoustic fields using spatial impulse responses. It is shown how both the pulsed emitted and scattered fields can be accurately derived using spatial impulse responses, and how attenuation and different boundary conditions can be incorporated. The chapter goes into some detail of deriving the different results and explaining their consequence. Different examples for both simulated and measured fields are given. The chapter is based on the papers [6], [7] and [8] and on the book [9].

2.1 Fields in linear acoustic systems

It is a well known fact in electrical engineering that a linear electrical system is fully characterized by its impulse response as shown in Fig. 2.1. Applying a delta function to the input of the circuit and measuring its output characterizes the system. The output $y(t)$ to any kind of input signal $x(t)$ is then given by

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\theta)x(t-\theta)d\theta, \tag{2.1}$$

where $h(t)$ is the impulse response of the linear system and $*$ denotes time convolution. The transfer function of the system is given by the Fourier transform of the impulse response and characterizes the systems amplification of a time-harmonic input signal.

The same approach can be taken to characterize a linear acoustic system. The basic set-up is shown in Fig. 2.2. The acoustic radiator (transducer) on the left is mounted in a infinite rigid, baffle and its position is denoted by $\vec{r}_2$. It radiates into a homogeneous medium with a constant speed of sound c and density ρ_0 throughout the medium. The point denoted by $\vec{r}_1$ is where the acoustic pressure from the transducer is measured by a small point hydrophone. A voltage excitation of the transducer with a delta function will give rise to a pressure field that is measured by the hydrophone. The measured response is the acoustic impulse response for this particular system with the given set-up. Moving the transducer or the hydrophone to a new position will give a different response. Moving the hydrophone closer to the

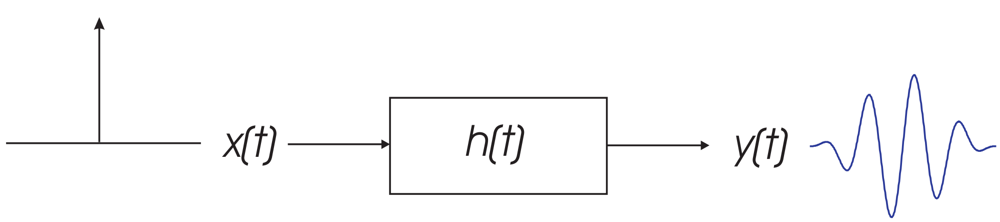


Figure 2.1: Measurement of impulse response for a linear electric system.

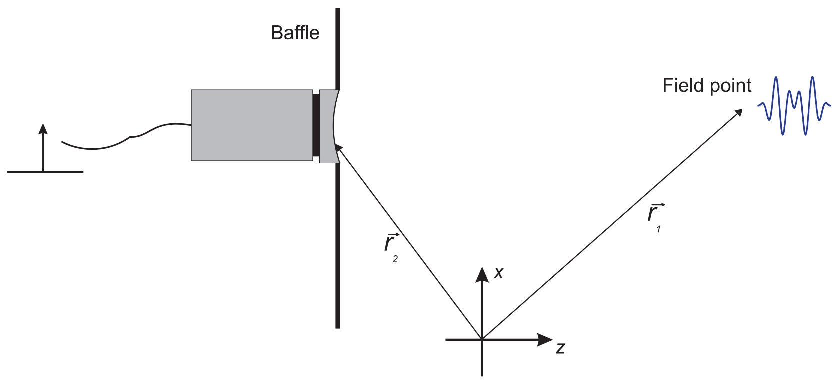


Figure 2.2: A linear acoustic system.

transducer surface will often increase the signal[1], and moving it away from the center axis of the transducer will often diminish it. Thus, the impulse response depends on the relative position of both the transmitter and receiver $(\vec{r}_2 - \vec{r}_1)$ and hence it is called a spatial impulse response.

A perception of the sound field for a fixed time instance can be obtained by employing Huygens' principle in which every point on the radiating surface is the origin of an outgoing spherical wave. This is illustrated in Fig. 2.3. Each of the outgoing spherical waves are given by

$$p_s(\vec{r}_1, t) = \delta\left(t - \frac{|\vec{r}_2 - \vec{r}_1|}{c}\right) = \delta\left(t - \frac{|r|}{c}\right) \tag{2.2}$$

where $\vec{r}_1$ indicates the point in space, $\vec{r}_2$ is the point on the transducer surface, and t is the time for the snapshot of the spatial distribution of the pressure. The spatial impulse response is then found by observing the pressure waves at a fixed position in space over time by having all the spherical waves pass the point of observation and summing them. Being on the acoustical axis of the transducer gives a short response whereas an off-axis point yields a longer impulse response as shown in Fig. 2.3.

2.2 Basic theory

In this section the exact expression for the spatial impulse response will more formally be derived. The basic setup is shown in Fig. 2.4. The triangularly shaped aperture is placed in an infinite, rigid baffle on which the velocity normal to the plane is zero, except at the aperture. The field point is denoted by $\vec{r}_1$ and the aperture by $\vec{r}_2$. The pressure field generated by the aperture is then found by the Rayleigh integral [10]

$$p(\vec{r}_1, t) = \frac{\rho_0}{2\pi} \int_S \frac{\dfrac{\partial v_n(\vec{r}_2, t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{\partial t}}{|\ \vec{r}_1 - \vec{r}_2\ |} dS, \tag{2.3}$$

where v_n is the velocity normal to the transducer surface. The integral is a statement of Huyghens' principle that the field is found by integrating the contributions from all the infinitesimally small area elements that make up the aperture. This integral formulation assumes linearity and propagation in a homogeneous medium without attenuation. Further, the radiating aperture is assumed flat, so no re-radiation from scattering and reflection takes place. Exchanging the integration and the partial derivative, the integral can be written as

$$p(\vec{r}_1, t) = \frac{\rho_0}{2\pi} \frac{\partial \displaystyle\int_S \frac{v_n(\vec{r}_2, t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{|\ \vec{r}_1 - \vec{r}_2\ |} dS}{\partial t}. \tag{2.4}$$

[1] This is not always the case. It depends on the focusing of the transducer. Moving closer to the transducer but away from its focus will decrease the signal.

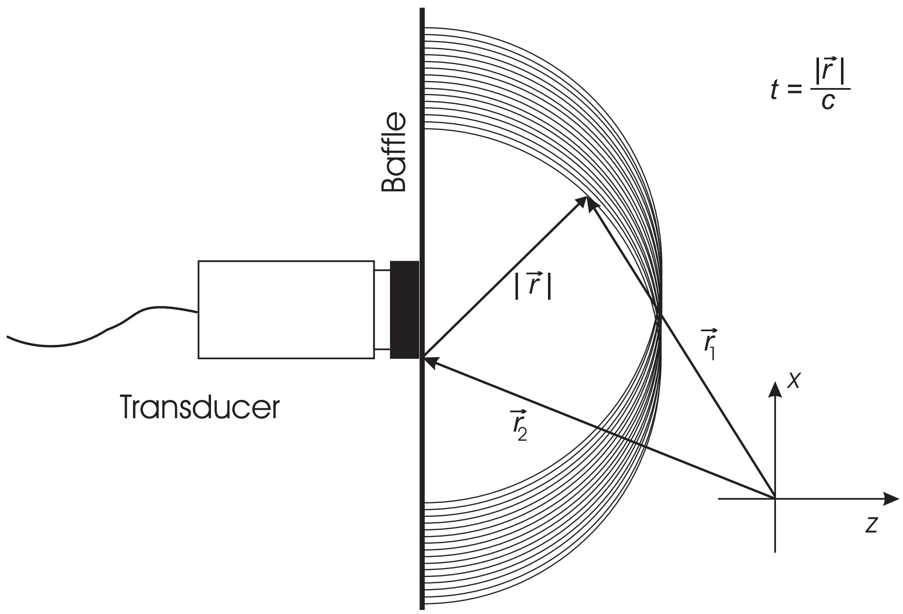


Figure 2.3: Illustration of Huygens' principle for a fixed time instance. A spherical wave with a radius of $|\vec{r}| = ct$ is radiated from each point on the aperture.

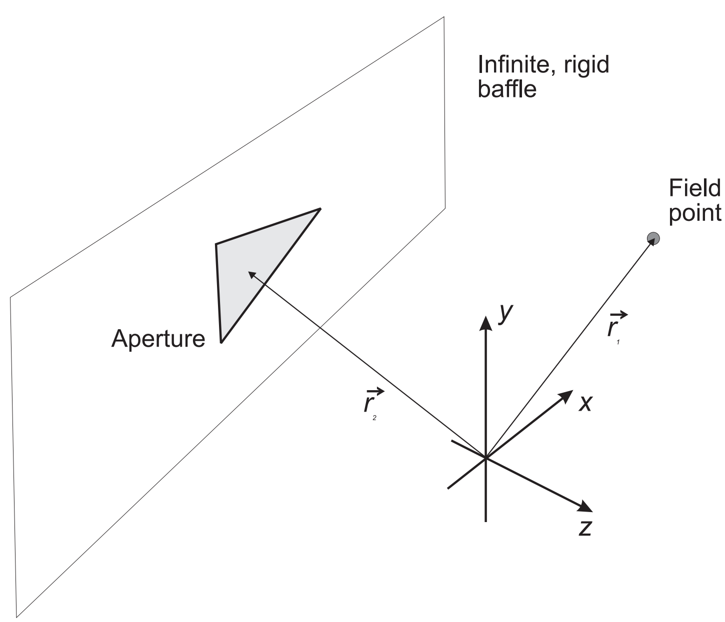


Figure 2.4: Position of transducer, field point, and coordinate system.

It is convenient to introduce the velocity potential ψ that satisfies the equations [11]

$$\begin{aligned}\vec{v}(\vec{r},t) &= -\nabla\psi(\vec{r},t)\\ p(\vec{r},t) &= \rho_0\frac{\partial\psi(\vec{r},t)}{\partial t}.\end{aligned} \tag{2.5}$$

Then only a scalar quantity need to be calculated and all field quantities can be derived from it. The surface integral is then equal to the velocity potential:

$$\psi(\vec{r}_1,t) = \int_S \frac{v_n(\vec{r}_2, t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS \tag{2.6}$$

The excitation pulse can be separated from the transducer geometry by introducing a time convolution with a delta function as

$$\psi(\vec{r}_1,t) = \int_S \int_T \frac{v_n(\vec{r}_2, t_2)\delta(t - t_2 - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dt_2 dS, \tag{2.7}$$

where δ is the Dirac delta function.

Assume now that the surface velocity is uniform over the aperture making it independent of $\vec{r}_2$, then:

$$\psi(\vec{r}_1,t) = v_n(t) * \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS, \tag{2.8}$$

where $*$ denotes convolution in time. The integral in this equation

$$h(\vec{r}_1,t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS \tag{2.9}$$

is called the spatial impulse response and characterizes the three-dimensional extent of the field for a particular transducer geometry. Note that this is a function of the relative position between the aperture and the field.

Using the spatial impulse response the pressure is written as

$$p(\vec{r}_1,t) = \rho_0 \frac{\partial v_n(t)}{\partial t} * h(\vec{r}_1,t) \tag{2.10}$$

which equals the emitted pulsed pressure for any kind of surface vibration $v_n(t)$. The continuous wave field can be found from the Fourier transform of (2.10). The received response for a collection of scatterers can also be found from the spatial impulse response [12], [6]. This is derived in Section 2.6. Thus, the calculation of the spatial impulse response makes it possible to find all ultrasound fields of interest.

2.2.1 Geometric considerations

The calculation of the spatial impulse response assumes linearity and any complex-shaped transducer can therefore be divided into smaller apertures and the response can be found by adding the responses from the sub-apertures. The integral is, as mentioned before, a statement of Huyghens' principle of summing contributions from all areas of the aperture.

An alternative interpretation is found by using the acoustic reciprocity theorem [13]. This states that: "If in an unchanging environment the locations of a small source and a small receiver are interchanged, the received signal will remain the same." Thus, the source and receiver can be interchanged. Emitting a spherical wave from the field point and finding the wave's intersection with the aperture also yields the spatial impulse response. The situation is depicted in Fig. 2.5, where an outgoing spherical wave is emitted from the origin of the coordinate system. The dashed curves indicate the circles from the projected spherical wave.

The calculation of the impulse response is then facilitated by projecting the field point onto the plane of the aperture. The task is thereby reduced to a two-dimensional problem and the field point is given as a (x, y) coordinate set and a

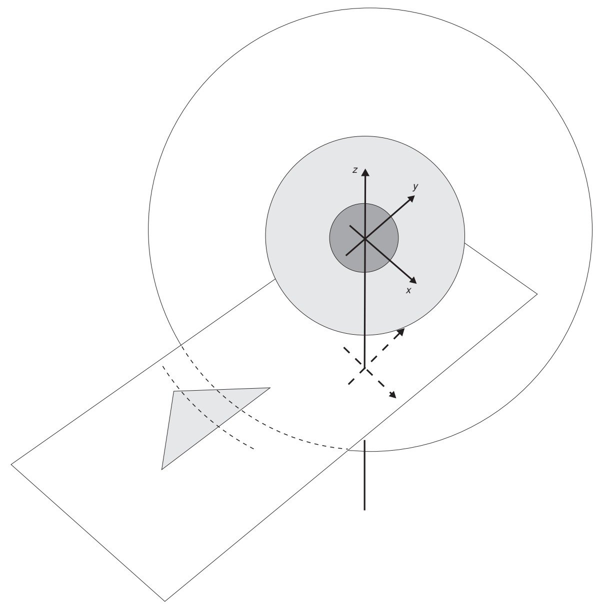


Figure 2.5: Emission of a spherical wave from the field point and its intersection of the aperture.

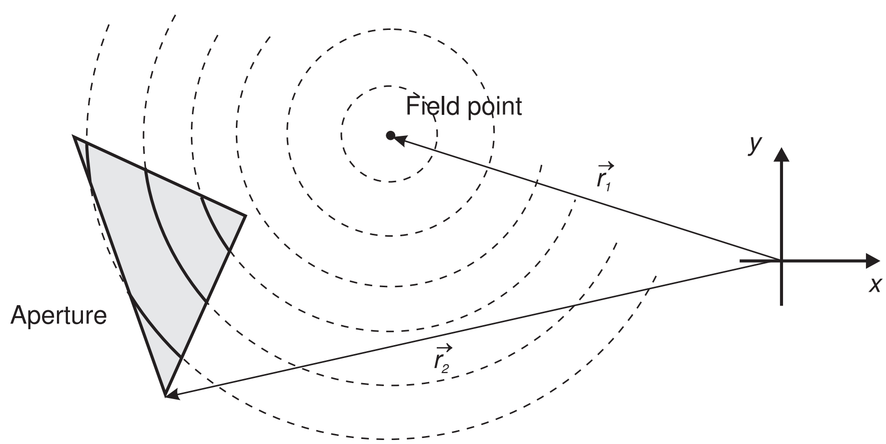


Figure 2.6: Intersection of spherical waves from the field point by the aperture, when the field point is projected onto the plane of the aperture.

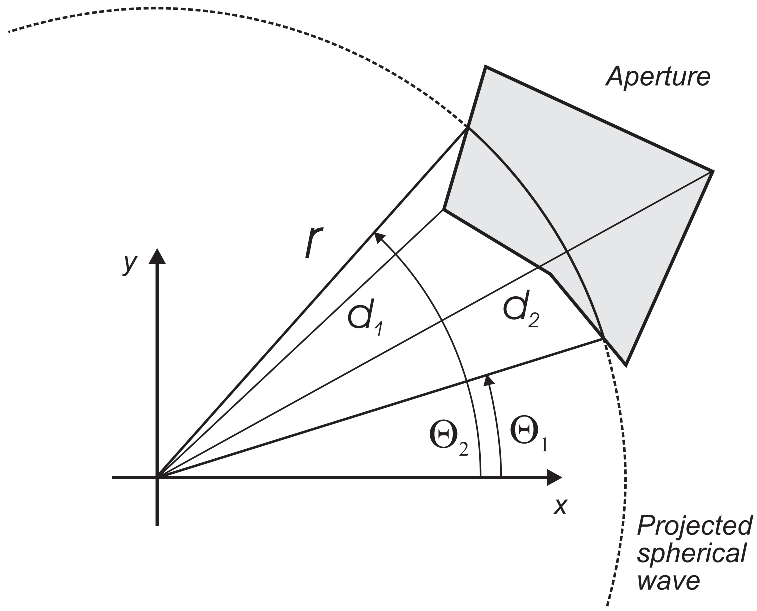


Figure 2.7: Definition of distances and angles in the aperture plan for evaluating the Rayleigh integral.

height z above the plane. The three-dimensional spherical waves are then reduced to circles in the $x-y$ plane with the origin at the position of the projected field point as shown in Fig. 2.6.

The spatial impulse response is, thus, determined by the relative length of the part of the arc that intersects the aperture. Thereby it is the crossing of the projected spherical waves with the edges of the aperture that determines the spatial impulse responses. This fact is used for deriving equations for the spatial impulse responses in the next section.

2.3 Calculation of spatial impulse responses

The spatial impulse response is found from the Rayleigh integral derived earlier

$$h(\vec{r}_1,t) = \int_S \frac{\delta(t-\frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi \mid \vec{r}_1-\vec{r}_2 \mid} dS \tag{2.11}$$

The task is to project the field point onto the plane coinciding with the aperture, and then find the intersection of the projected spherical wave (the circle) with the active aperture as shown in Fig. 2.6.

Rewriting the integral into polar coordinates gives:

$$h(\vec{r}_1,t) = \int_{\Theta_1}^{\Theta_2} \int_{d_1}^{d_2} \frac{\delta(t-\frac{R}{c})}{2\pi R} r\, dr\, d\Theta \tag{2.12}$$

where r is the radius of the projected circle and R is the distance from the field point to the aperture given by $R^2 = r^2 + z_p^2$. Here z_p is the field point height above the $x-y$ plane of the aperture. The projected distances d_1, d_2 are determined by the aperture and are the distance closest to and furthest away from the aperture, and Θ_1, Θ_2 are the corresponding angles for a given time (see Fig. 2.7).

Introducing the substitution $2RdR = 2rdr$ gives

$$h(\vec{r}_1,t) = \frac{1}{2\pi} \int_{\Theta_1}^{\Theta_2} \int_{R_1}^{R_2} \delta(t-\frac{R}{c})\, dR\, d\Theta \tag{2.13}$$

The variables R_1 and R_2 denote the edges closest to and furthest away from the field point. Finally using the substitution $t' = R/c$ gives

$$h(\vec{r}_1, t) = \frac{c}{2\pi} \int_{\Theta_1}^{\Theta_2} \int_{t_1}^{t_2} \delta(t - t') dt' d\Theta \tag{2.14}$$

For a given time instance the contribution along the arc is constant and the integral gives

$$h(\vec{r}_1, t) = \frac{\Theta_2 - \Theta_1}{2\pi} c \tag{2.15}$$

when assuming the circle arc is only intersected once by the aperture. The angles Θ_1 and Θ_2 are determined by the intersection of the aperture and the projected spherical wave, and the spatial impulse response is, thus, solely determined by these intersections, when no apodization of the aperture is used. The response can therefore be evaluated by keeping track of the intersections as a function of time.

2.3.1 A simple calculation procedure

From the derivation in the last section it can be seen that the spatial impulse response in general can be expressed as

$$h(\vec{r}_1, t) = \frac{c}{2\pi} \sum_{i=1}^{N(t)} \left[\Theta_2^{(i)}(t) - \Theta_1^{(i)}(t) \right] \tag{2.16}$$

where $N(t)$ is the number of arc segments that crosses the boundary of the aperture for a given time and $\Theta_2^{(i)}(t), \Theta_1^{(i)}(t)$ are the associated angles of the arc. This was also noted by Stepanishen [14]. The calculation can, thus, be formulated as finding the angles of the aperture edge's intersections with the projected spherical wave, sorting the angles, and then summing the arc angles that belong to the aperture. Finding the intersections can be done from the description of the edges of the aperture. A triangle can be described by three lines, a rectangle by four, and the intersections are then found from the intersections of the circle with the lines. This makes it possible to devise a general procedure for calculating spatial impulse responses for any flat, bounded aperture, since the task is just to find the intersections of the boundary with the circle.

The spatial impulse response is calculated from the time the aperture first is intersected by a spherical wave to the time for the intersection furthest away. The intersections are found for every time instance and the corresponding angles are sorted. The angles lie in the interval from 0 to 2π. It is then found whether the arc between two angles belongs to the aperture, and the angle difference is added to the sum, if the arc segment is inside the aperture. This yields the spatial impulse response according to Eq. (2.16). The approach can be described by the flow chart shown in Fig. 2.8.

The only part of the algorithm specific to the aperture is the determination of the intersections and whether the point is inside the aperture. Section 2.3.2 shows how this is done for polygons, Section 2.3.3 for circles, and Section 2.3.6 for higher-order parametric boundaries.

All the intersections need not be found for all times. New intersections are only introduced, when a new edge or corner of the aperture is met. Between times when two such corners or edges are encountered the number of intersections remains constant and only intersections, which belong to points inside the aperture need to be found. Note that an aperture edge gives rise to a discontinuity in the spatial impulse response. Also testing whether the point is inside the aperture is often superfluous, since this only needs to be found once after each discontinuity in the response. These two observations can significantly reduce the number of calculations, since only the intersections affecting the response are found.

The procedure first finds the number of discontinuities. Then only intersection influencing the response are calculated between two discontinuity points. This can potentially make the approach faster than the traditional approach, where the response from a number of different rectangles or triangles must be calculated.

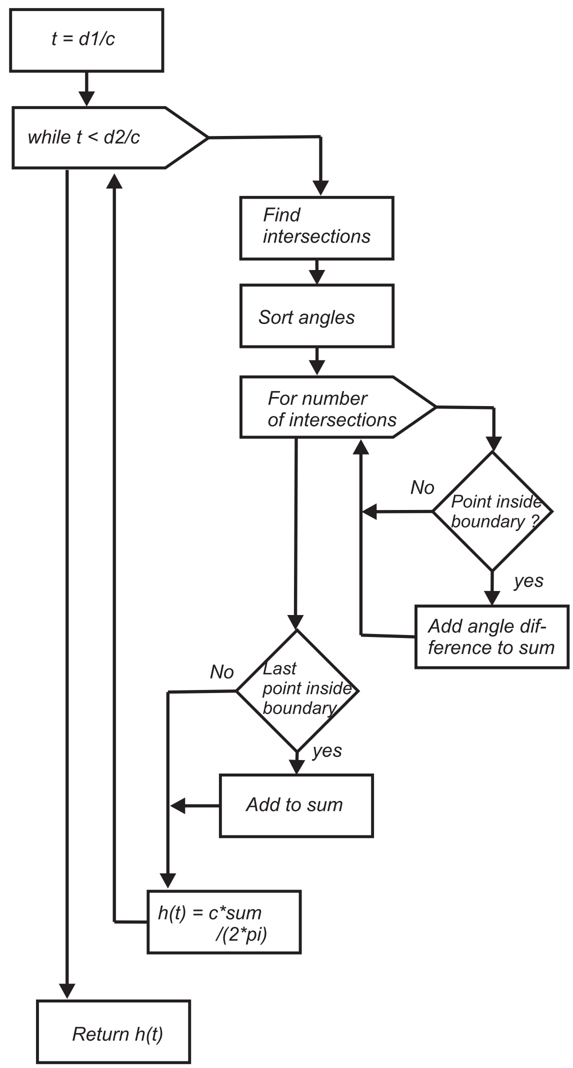


Figure 2.8: Flow chart for calculating the spatial impulse response.

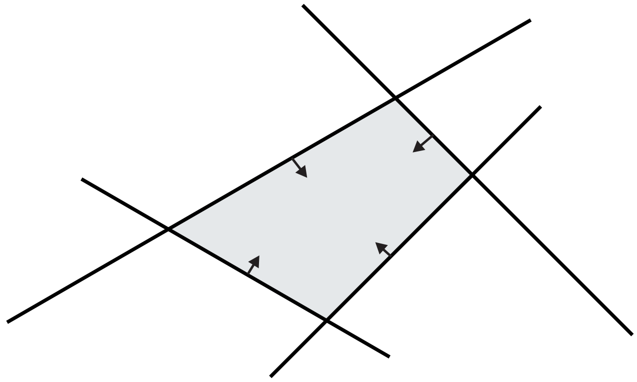


Figure 2.9: Definition of bounding lines for polygon transducer. The arrows indicate the half-planes for the active aperture.

2.3.2 Solution for polygons

The boundary of any polygon can be defined by a set of bounding lines as shown in Fig. 2.9.

The active aperture is then defined as lying on one side of the line as indicated by the arrows, and a point on the aperture must be placed correctly in relation to all lines. The test whether a point is on the aperture is thus to go through all lines and test whether the point lies in the active half space for the line, and stop if it is not. The point is inside the aperture, if it passes the test for all the lines.

The intersections are found from the individual intersections between the projected circle and the lines. They are determined from the equations for the projected spherical wave and the line:

$$\begin{aligned} r^2 &= (x-x_0)^2 + (y-y_0)^2 \\ y &= \alpha x + y_1 \\ r^2 &= (ct)^2 - z_p^2 \end{aligned} \tag{2.17}$$

Here (x_0, y_0) is the center of the circle, α the slope of the line, and y_1 its intersect with the y-axis. The intersections are given from the solutions to:

$$\begin{aligned} 0 &= (1+\alpha^2)x^2 + (2\alpha y_1 - 2x_0 - 2y_0\alpha)x + (y_0^2 + y_1^2 + x_0^2 - 2y_0 y_1 - r^2) \\ &= Ax^2 + Bx + C \\ D &= B^2 - 4AC \end{aligned} \tag{2.18}$$

The angles are

$$\Theta = \arctan\left(\frac{y-y_0}{x-x_0}\right) \tag{2.19}$$

Intersections between the line and the circle are only found if $D > 0$. A determinant $D < 0$ indicates that the circle did not intersect the line. If the line has infinite slope, the solution is found from the equation:

$$\begin{aligned} x &= x_1 \\ 0 &= y^2 - 2y_0 y + y_0^2 + (x_1 - x_0)^2 - r^2 \\ &= A_\infty y^2 + B_\infty y + C_\infty \end{aligned} \tag{2.20}$$

in which $A_\infty, B_\infty, C_\infty$ replace A, B, C, respectively, and the solutions are found for y rather than x. Here x_1 is the line's intersection with the x-axis.

The times for discontinuities in the spatial impulse response are given by the intersections of the lines that define the aperture's edges and by the minimum distance from the projected field point to the lines. The minimum distance is found from a line passing through the field point that is orthogonal to the bounding line. The intersection between the orthogonal line and the bounding line is:

$$
\begin{aligned}
x &= \frac{\alpha y_p + x_p - \alpha y_1}{\alpha^2 + 1} \\
y &= \alpha x + y_1
\end{aligned}
\tag{2.21}
$$

where (x_p, y_p, z_p) is the position of the field point. For an infinite slope line the solution is $x = x_1$ and $y = y_p$. The corresponding time is:

$$
t_i = \frac{\sqrt{(x - x_p)^2 + (y - y_p)^2 + z_p^2}}{c} \tag{2.22}
$$

The intersections of the lines are also found, and the corresponding times are calculated by (2.22) and sorted in ascending order. They indicate the start and end time for the response and the time points for discontinuities in the response.

2.3.3 Solution for circular surfaces

The other basic shape for a transducer apart from rectangular shapes is the flat, round surface used for single element piston transducers and annular arrays. For these the intersections are determined by two circles as depicted in Fig. 2.10.

Here O_1 is the center of the aperture with radius r_a and the projected spherical wave is centered at O_2 with radius $r_b(t) = \sqrt{(ct)^2 - z_p^2}$. The length $h_a(t)$ is given by [15, page 66]

$$
\begin{aligned}
h_a(t) &= \frac{2\sqrt{p(t)(p(t) - a)(p(t) - r_a)(p(t) - r_b(t))}}{a} \\
a &= ||O_1 - O_2|| \\
p(t) &= \frac{a + r_a + r_b(t)}{2}
\end{aligned}
\tag{2.23}
$$

In a coordinate system centered at O_1 and an x-axis in the $O_1 - O_2$ direction, the intersections are at

$$
\begin{aligned}
y &= h_a(t) \\
l &= \pm\sqrt{r_b^2(t) - h_a^2(t)}
\end{aligned}
\tag{2.24}
$$

The sign for l depends on the position of the intersections. A negative sign is used if the intersections are for negative values of x, and positive sign is used for positive x positions.

When the field point is outside the active aperture the spatial impulse response is

$$
\begin{aligned}
h(\vec{r}_1, t) &= \frac{|\Theta_2 - \Theta_1|}{2\pi} c = \frac{c}{\pi} \arctan\left(\frac{h_a(t)}{l}\right) \\
\Theta_2 &= \arctan\left(\frac{h_a(t)}{l}\right) = -\Theta_1
\end{aligned}
\tag{2.25}
$$

It must be noted that a proper four-quadrant arc-tan should be used to give the correct response. An alternative formula is [16, page 19]

$$
\begin{aligned}
h(\vec{r}_1, t) &= \frac{c}{2\pi} \arcsin\left(\frac{2\sqrt{p(t)(p(t) - a)(p(t) - r_a)(p(t) - r_b(t))}}{r_b^2(t)}\right) \\
&= \frac{c}{2\pi} \arcsin\left(\frac{a h_a(t)}{r_b^2(t)}\right)
\end{aligned}
\tag{2.26}
$$

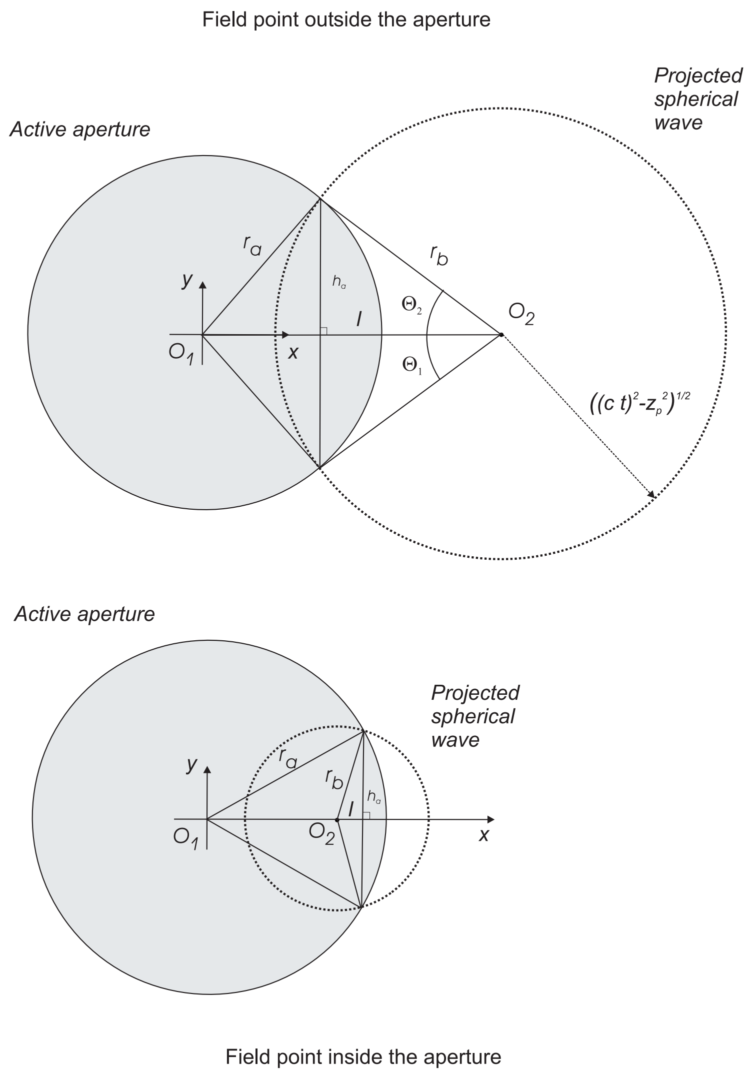


Figure 2.10: Geometry for determining intersections between circles. The top graph shows the geometry when the field point denoted by O_2 is outside the aperture, and the bottom graph when it is inside.

The start time t_s for the response is found from

$$\begin{aligned} r_a + r_b(t) &= ||O_1 - O_2|| \\ t_s &= \frac{\sqrt{r_b^2(t) + z_p^2}}{c} = \frac{\sqrt{(||O_1 - O_2|| - r_a)^2 + z_p^2}}{c} \end{aligned} \quad (2.27)$$

and the response ends at the time t_e when

$$\begin{aligned} r_b(t) &= r_a + ||O_1 - O_2|| \\ t_e &= \frac{\sqrt{r_b^2(t) + z_p^2}}{c} = \frac{\sqrt{(||O_1 - O_2|| + r_a)^2 + z_p^2}}{c} \end{aligned} \quad (2.28)$$

When the field point is inside the aperture, the response is

$$h(\vec{r}_1, t) = c \quad \text{for} \quad \frac{z_p}{c} \le t \le \frac{\sqrt{(r_a - ||O_1 - O_2||)^2 + z_p^2}}{c} \quad (2.29)$$

thereafter the arc lying outside the aperture should be subtracted, so that

$$h(\vec{r}_1, t) = \frac{2\pi - |\Theta_2 - \Theta_1|}{2\pi} c \quad (2.30)$$

The response ends when

$$\begin{aligned} r_b(t) &= r_a + ||O_1 - O_2|| \\ t_e &= \frac{\sqrt{(||O_1 - O_2|| + r_a)^2 + z_p^2}}{c} \end{aligned} \quad (2.31)$$

The determination of which part of the arc that subtracts or adds to the response is determined by what the active aperture is. One ring in an annular array can be defined as consisting of an active aperture outside a circle combined with an active aperture inside a circle for defining the inner and outer rim of the aperture. A circular aperture can also be combined with a line for defining the active area of a split aperture used for continuous wave probing.

2.3.4 Solution for a circular concave surface

For reference the expression for a concave transducer, which is a type often used in medical ultrasonics, is given. A derivation of the solution can be found in [17] and [18].

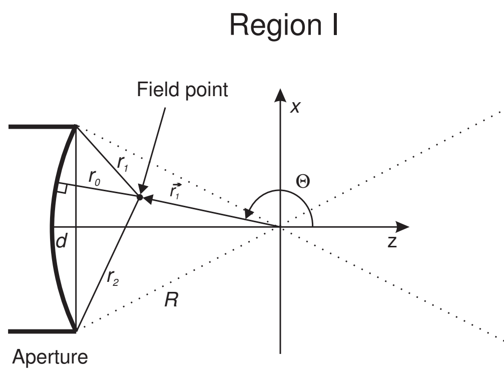
The spatial impulse response is [17]:

$$h_c(\vec{r}_1, t) = \left\{ \begin{array}{l|c|c|c|} & \multicolumn{2}{c|}{\text{Region I}} & \text{Region II} \\ & \underline{z < 0} & \underline{z > 0} & \\ 0 & ct < r_0 & r_0 < ct & ct < r_1 \\ \frac{cR}{r} & r_0 < ct < r_1 & r_2 < ct < r_0 & - \\ \frac{cR}{r}\frac{1}{\pi}\arccos\frac{\eta(t)}{\sigma(t)} & r_1 < ct < r_2 & r_1 < ct < r_2 & r_1 < ct < r_2 \\ 0 & r_2 < ct & ct < r_1 & r_2 < ct \end{array} \right. \quad (2.32)$$

where:

$$\begin{aligned} \eta(t) &= R\left\{\frac{1 - d/R}{\sin\Theta} + \frac{1}{\tan\Theta}\left(\frac{R^2 + r^2 - c^2t^2}{2rR}\right)\right\} \\ \sigma(t) &= R\sqrt{1 - \left(\frac{R^2 + r^2 - c^2t^2}{2rR}\right)^2} \\ r &= |\vec{r}_1|. \end{aligned} \quad (2.33)$$

The variables are defined in Fig. 2.11.



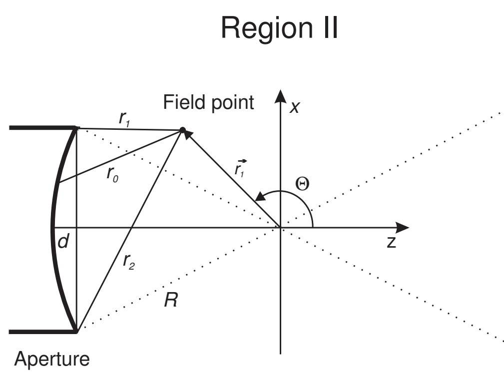


Figure 2.11: Definition of variables for spatial impulse response of concave transducer.

2.3.5 Solution for other surfaces

Analytical solutions for spatial impulse response have been derived for a number of different geometries by various authors. The response from a flat rectangle can be found in [19, 20], and for a flat triangle in [21]).

2.3.6 Solution for parametric surfaces

For ellipses or other higher order parametric surfaces it is in general not easy to find analytic solutions for the spatial impulse response. The boundary method described above can, however, be used for providing a simple solution to the problem, since the intersections between the projected spherical wave and the edge of the aperture uniquely determine the spatial impulse response. It is therefore possible to use root finding for a set of (non-linear) equations for finding these intersections. The problem is to find when both the spherical wave and the aperture have crossing contours in the plane of the aperture, *i.e.*, when

$$\begin{aligned} (ct)^2 - z_p^2 - (x-x_p)^2 - (y-y_p)^2 &= 0 \\ S(x,y) &= 0 \end{aligned} \tag{2.34}$$

in which $S(x,y) = 0$ defines the boundary of the aperture. The problem of numerically finding these roots is in general not easy, if a good initial guess on the position of the intersections is not found [22, pages 286–289]. Good initial values are however found here, since the intersections must lie on the projected circle and the intersections only move slightly from time point to time point. An efficient Newton-Raphson algorithm can therefore be devised for finding the intersections, and the procedure detailed here can be used to find the spatial impulse response for any flat transducer geometry with an arbitrary apodization and both hard and soft baffle mounting.

2.4 Apodization and soft baffle

Often ultrasound transducers do not vibrate as a piston over the aperture. This can be due to the clamping of the active surface at its edges, or intentionally to reduce side-lobes in the field. Applying for example a Gaussian apodization will significantly lower side lobes and generate a field with a more uniform point spread function as a function of depth. Apodization is introduced in (2.12) by writing [23]

$$h(\vec{r}_1,t) = \int_{\Theta_1}^{\Theta_2}\int_{d_1}^{d_2} a_p(r,\Theta)\frac{\delta(t-\frac{R}{c})}{2\pi R} r\,dr\,d\Theta \tag{2.35}$$

in which $a_p(r,\Theta)$ is the apodization over the aperture. Using the same substitutions as before yields

$$h(\vec{r}_1,t) = \frac{c}{2\pi}\int_{\Theta_1}^{\Theta_2}\int_{t_1}^{t_2} a_{p1}(t',\Theta)\delta(t-t')\,dt'\,d\Theta \tag{2.36}$$

where $a_{p1}(t',\Theta) = a_p(\sqrt{(ct')^2 - z_p^2},\Theta)$. The inner integral is a convolution of the apodization function with a δ-function and readily yields

$$h(\vec{r}_1,t) = \frac{c}{2\pi}\int_{\Theta_1}^{\Theta_2} a_{p1}(t,\Theta)\,d\Theta \tag{2.37}$$

as noted by several authors [23, 24, 25]. The response for a given time instance can, thus, be found by integrating the apodization function along the fixed arc with a radius of $r = \sqrt{(ct)^2 - z_p^2}$ for the angles for the active aperture. Any apodization function can therefore be incorporated into the calculation by employing numerical integration.

Often the assumption of an infinite rigid baffle for the transducer mounting is not appropriate and another form of the Rayleigh integral must be used. For a soft baffle, in which the pressure on the baffle surface is zero, the Rayleigh-Sommerfeld integral is used. This is [26, pages 46–50]

$$h_s(\vec{r}_1,t) = \int_S \frac{\delta(t-\frac{|\vec{r}_1-\vec{r}_2|}{c})}{2\pi\mid\vec{r}_1-\vec{r}_2\mid}\cos\varphi\,dS \tag{2.38}$$

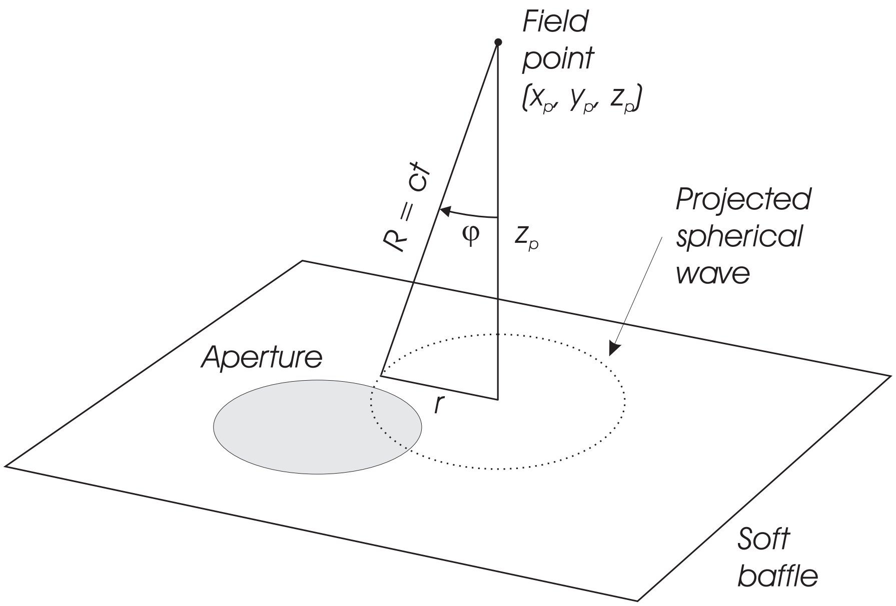


Figure 2.12: Definition of angle used for a soft baffle.

assuming that $|\vec{r}_1 - \vec{r}_2| \gg \lambda$. Here $\cos\varphi$ is the angle between the line through the field point orthogonal to the aperture plane and the radius of the spherical wave as shown in Fig. 2.12.

The angle φ is constant for a given radius of the projected spherical wave and thus for a given time. It is given by

$$\cos\varphi = \frac{z_p}{R} = \frac{z_p}{ct} \tag{2.39}$$

Using the substitutions from Section 2.2, the Rayleigh-Sommerfeld integral can then be rewritten as

$$h_s(\vec{r}_1, t) = \frac{z_p}{2\pi} c(\Theta_2 - \Theta_1) \int_{t_1}^{t_2} \frac{\delta(t - t')}{ct'} \, dt' \tag{2.40}$$

Using the property of the δ-function that

$$\int_{-\infty}^{+\infty} g(t')\delta(t - t') \, dt' = g(t) \tag{2.41}$$

then gives

$$h_s(\vec{r}_1, t) = \frac{z_p}{ct} \frac{\Theta_2 - \Theta_1}{2\pi} c = \frac{z_p}{ct} h(\vec{r}_1, t). \tag{2.42}$$

The spatial impulse response can, thus, be found from the spatial impulse response for the rigid baffle case by multiplying with $z_p/(ct)$.

2.5 Examples of spatial impulse responses

The first example shows the spatial impulse responses from a 3×5 mm rectangle for different spatial positions 5 mm from the front face of the transducer. The responses are found from the center of the rectangle $(y = 0)$ and out in steps of 2 mm in the x direction to 6 mm away from the center of the rectangle. A schematic diagram of the situation is shown in Fig. 2.13 for the on-axis response. The impulse response is zero before the first spherical wave reaches the aperture. Then the response stays constant at a value of c. The first edge of the aperture is met, and the response drops

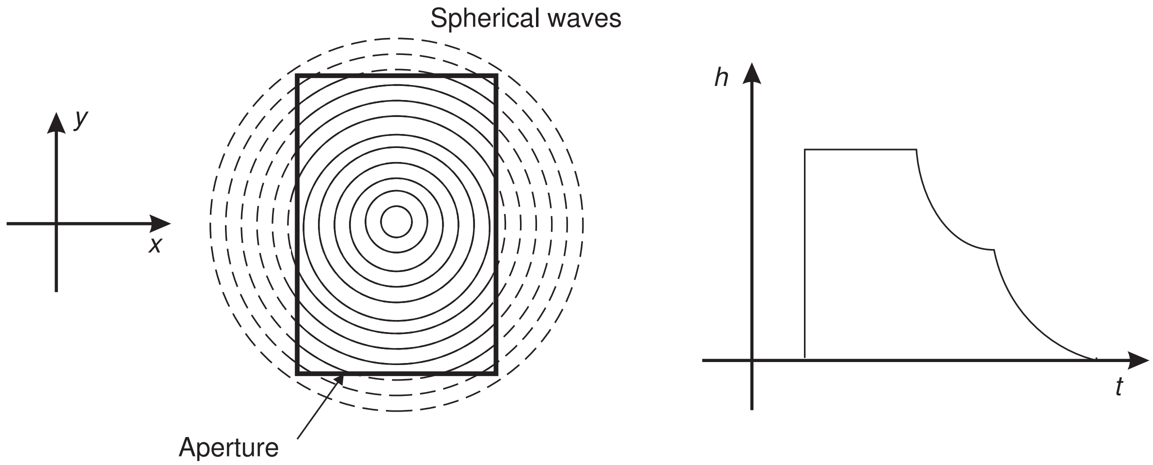


Figure 2.13: Schematic diagram of field from rectangular element.

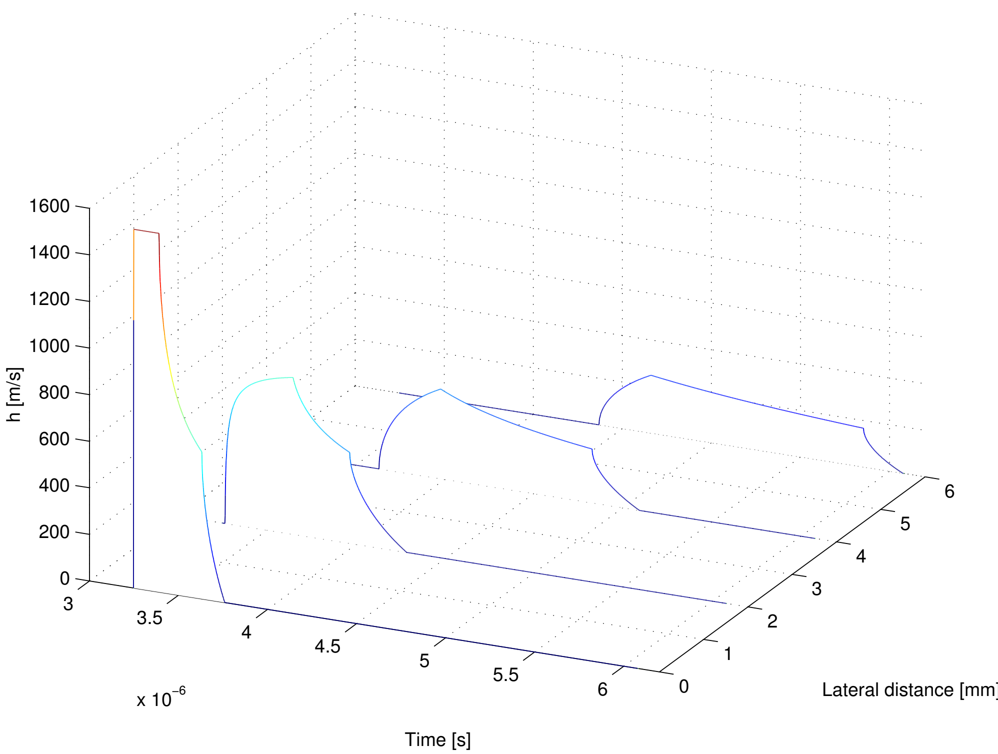


Figure 2.14: Spatial impulse response from a rectangular aperture of 4×5 mm at for different lateral positions

of. The decrease with time in increased when the next edge of the aperture is reached and the response becomes zero when the projected spherical waves all are outside the area of the aperture.

A plot of the results for the different lateral field positions is shown in Fig. 2.14. It can be seen how the spatial impulse response changes as a function of relative position to the aperture.

The second example shows the response from a circular, flat transducer. Two different cases are shown in Fig. 2.15. The top graph shows the traditional spatial impulse response when no apodization is used, so that the aperture vibrates as a piston. The field is calculated 10 mm from the front face of the transducer starting at the center axis of the aperture. Twenty-one responses for lateral distance of 0 to 20 mm off axis are then shown. The same calculation is repeated in the bottom graph, when a Gaussian apodization has been imposed on the aperture. The vibration amplitude is a factor of $1/\exp(4)$ less at the edges of the aperture than at the center. It is seen how the apodization reduces some of the sharp discontinuities in the spatial impulse response, which can reduce the sidelobes of the field.

2.6 Calculation of the scattered signal

In medical ultrasound, a pulsed field is emitted into the body and is scattered and reflected by density and propagation velocity perturbations. The scattered field then propagates back through the tissue and is received by the transducer. The field is converted to a voltage signal and used for the display of the ultrasound image. A full description of a typical imaging system, using the concept of spatial impulse response, is the purpose of the section.

The received signal can be found by solving an appropriate wave equation. This has been done in a number of papers (e.g. [27], [28]). Gore and Leeman [27] considered a wave equation where the scattering term was a function of the adiabatic compressibility and the density. The transducer was modeled by an axial and lateral pulse that were separable. Fatemi and Kak [28] used a wave equation where scattering originated only from velocity fluctuations, and the transducer was restricted to be circularly symmetric and unfocused (flat).

The scattering term for the wave equation used here is a function of density and propagation velocity perturbations, and the wave equation is equivalent to the one used by Gore and Leeman [27]. No restrictions are enforced on the transducer geometry or its excitation, and analytic expressions for a number of geometries can be incorporated into the model.

The model includes attenuation due to propagation and scattering, but not the dispersive attenuation observed for propagation in tissue. This can, however, be incorporated into the model as indicated in Section 2.6.6.

The derivation is organized as follows. The following section derives the wave equation and describes the different linearity assumptions made. Section 2.6.2 calculates the scattered field and section 2.6.3 introduces the spatial impulse response model for the incident field. Section 2.6.4 combines the wave equation solution and the transducer model to give the final equation for the received pressure field. To indicate the precision of the model, a single example of a predicted pressure field compared to measured field is given in Section 2.6.5.

2.6.1 Derivation of the wave equation

This section derives the wave equation. The section has been included in order to explain in detail the different linearity assumptions and approximations made to obtain a solvable wave equation. The derivation closely follows that developed by Chernov (1960).

The first approximation states that the instantaneous acoustic pressure and density can be written

$$P_{ins}(\vec{r},t) = P + p_1(\vec{r},t) \quad (2.43)$$

$$\rho_{ins}(\vec{r},t) = \rho(\vec{r}) + \rho_1(\vec{r},t) \quad (2.44)$$

in which P is the mean pressure of the medium and ρ is the density of the undisturbed medium. Here p_1 is the pressure

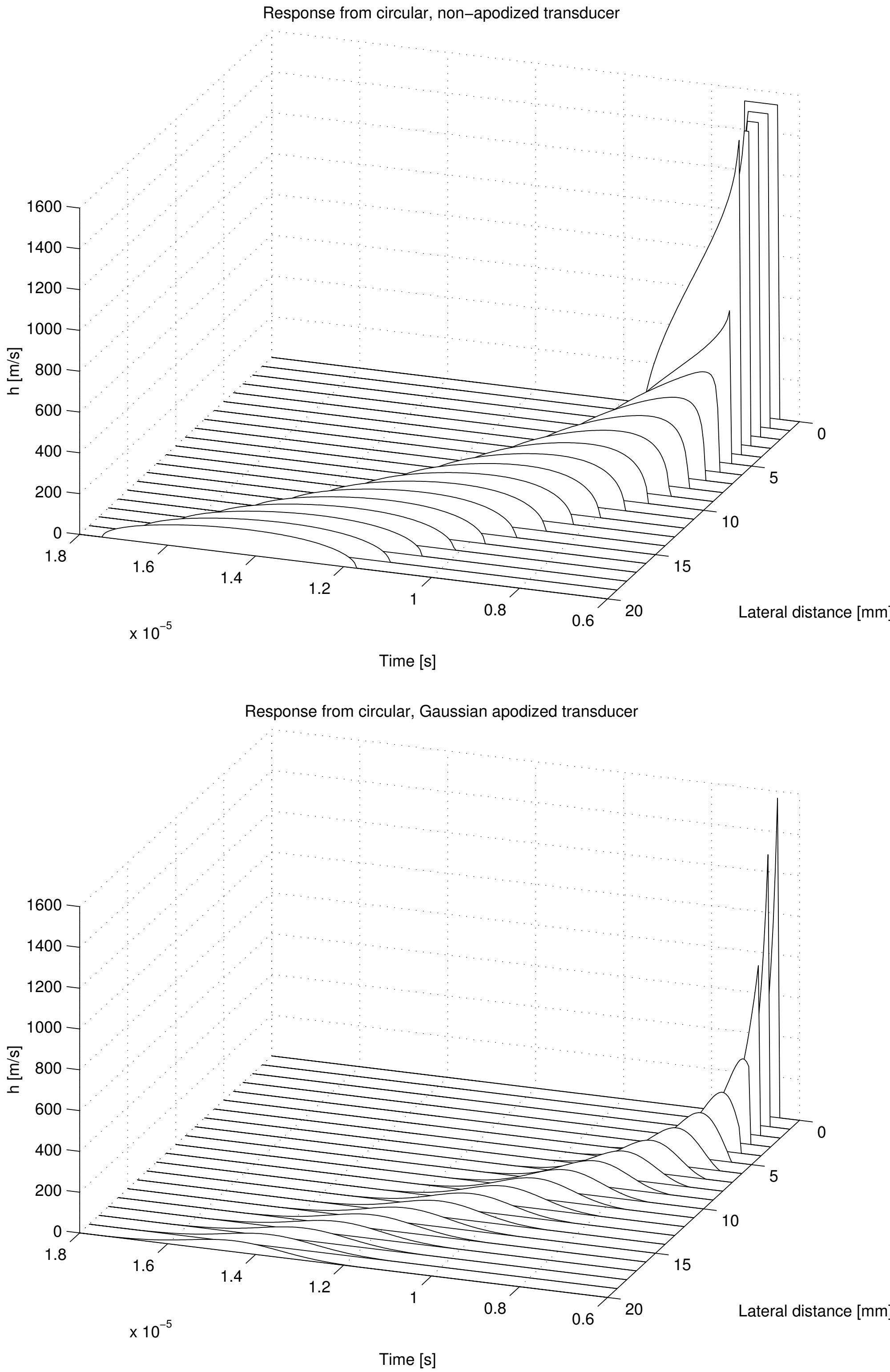


Figure 2.15: Spatial impulse response from a circular aperture. Graphs are shown without apodization of the aperture (top) and with a Gaussian apodization function (bottom). The radius of the aperture is 5 mm and the field is calculated 10 mm from the transducer surface.

variation caused by the ultrasound wave and is considered small compared to P and ρ_1 is the density change caused by the wave. Both p_1 and ρ_1 are small quantities of first order.

Our second assumption is that no heat conduction or conversion of ultrasound to thermal energy take place. Thus, the entropy is constant for the process, so that the acoustic pressure and density satisfy the adiabatic equation [29]:

$$\frac{dP_{ins}}{dt} = c^2 \frac{d\rho_{ins}}{dt} \tag{2.45}$$

The equation contains total derivatives, as the relation is satisfied for a given particle of the tissue rather than at a given point in space. This is the Lagrange description of the motion [11]. For our purpose the Euler description is more appropriate. Here the coordinate system is fixed in space and the equation describes the properties of whatever particle of fluid there is at a given point at a given time. Converting to an Eulerian description results in the following constitutive equation [29], [11]:

$$\frac{1}{c^2}\frac{\partial p_1}{\partial t} = \frac{\partial \rho_1}{\partial t} + \vec{u} \cdot \nabla \rho \tag{2.46}$$

using that P and ρ do not depend on time and that ρ_1 is small compared to ρ. Here u is the particle velocity, ∇ is the gradient operator, and $\cdot$ symbolizes the scalar product.

The pressure, density, and particle velocity must also satisfy the hydrodynamic equations [29]:

$$\rho_{ins}\frac{d\vec{u}}{dt} = -\nabla P_{ins} \tag{2.47}$$

$$\frac{\partial \rho_{ins}}{\partial t} = -\nabla \cdot (\rho_{ins}\vec{u}) \tag{2.48}$$

which are the dynamic equation and the equation of continuity. Using (2.43) and (2.44) and discarding higher order terms we can write

$$\rho\frac{\partial \vec{u}}{\partial t} = -\nabla p_1 \tag{2.49}$$

$$\frac{\partial \rho_1}{\partial t} = -\nabla \cdot (\rho\vec{u}) \tag{2.50}$$

Differentiating (2.50) with respect to t and inserting (2.49) gives

$$\frac{\partial^2 \rho_1}{\partial^2 t} = -\nabla \cdot (\rho\frac{\partial \vec{u}}{\partial t}) = -\nabla \cdot (-\nabla p_1) = \nabla^2 p_1 \tag{2.51}$$

Differentiating (2.46) with respect to t

$$\frac{1}{c^2}\frac{\partial^2 p_1}{\partial^2 t} = \frac{\partial^2 \rho_1}{\partial^2 t} + \frac{\partial \vec{u}}{\partial t} \cdot \nabla \rho \tag{2.52}$$

and inserting (2.51) and (2.49) leads to

$$\nabla^2 p_1 - \frac{1}{c^2}\frac{\partial^2 p_1}{\partial^2 t} = \frac{1}{\rho}\nabla \rho \cdot \nabla p_1 \tag{2.53}$$

Assuming that the propagation velocity and the density only vary slightly from their mean values yields

$$\begin{aligned} \rho(\vec{r}) &= \rho_0 + \Delta\rho(\vec{r}) \\ c(\vec{r}) &= c_0 + \Delta c(\vec{r}) \end{aligned} \tag{2.54}$$

where $\rho_0 \gg \Delta\rho$ and $c_0 \gg \Delta c$.

$$\nabla^2 p_1 - \frac{1}{(c_0 + \Delta c)^2}\frac{\partial^2 p_1}{\partial^2 t} = \frac{1}{(\rho_0 + \Delta\rho)}\nabla(\rho_0 + \Delta\rho) \cdot \nabla p_1 \tag{2.55}$$

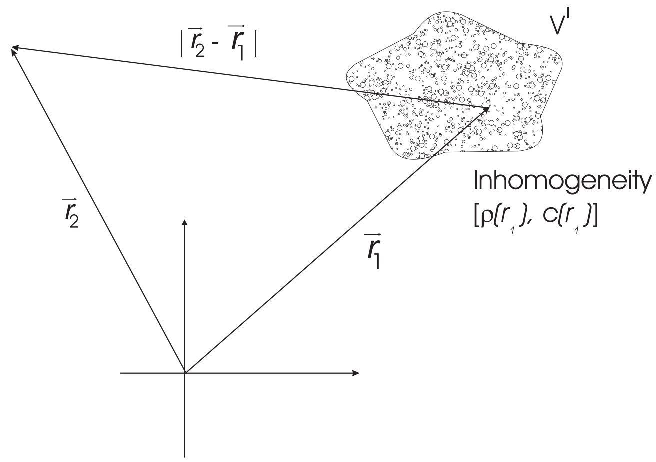


Figure 2.16: Coordinate system for calculating the scattered field

Ignoring small quantities of second order and using the approximation ($\Delta \ll 1$):

$$\frac{1}{1+\Delta} \approx 1 - \Delta \tag{2.56}$$

gives:

$$\nabla^2 p_1 - (\frac{1}{{c_0}^2} - \frac{2\Delta c}{{c_0}^3})\frac{\partial^2 p_1}{\partial^2 t} = \left(\frac{1}{\rho_0}\nabla(\Delta\rho) - \frac{\Delta\rho}{{\rho_0}^2}\nabla(\Delta\rho)\right) \cdot \nabla p_1 \tag{2.57}$$

Neglecting the second order term $(\Delta\rho/{\rho_0}^2)\nabla(\Delta\rho) \cdot \nabla p_1$ yields the wave equation:

$$\nabla^2 p_1 - \frac{1}{c_0^2}\frac{\partial^2 p_1}{\partial t^2} = -\frac{2\Delta c}{c_0^3}\frac{\partial^2 p_1}{\partial t^2} + \frac{1}{\rho_0}\nabla(\Delta\rho) \cdot \nabla p_1 \tag{2.58}$$

The two terms on the right side of the equation are the scattering terms which vanish for a homogeneous medium. The wave equation was derived by Chernov [29]. It has also been considered in Gore & Leeman [27] and Morse & Ingard [11] in a slightly different form, where the scattering terms were a function of the adiabatic compressibility κ and the density.

2.6.2 Calculation of the scattered field

Having derived a suitable wave equation, we now calculate the scattered field from a small inhomogeneity embedded in a homogeneous surrounding. The scene is depicted in Fig. 2.16. The inhomogeneity is identified by $\vec{r}_1$ and enclosed in the volume V'. The scattered field is calculated at the point indicated by $\vec{r}_2$ by integrating all the spherical waves emanating from the scattering region V' using the time dependent Green's function for unbounded space. Thus, the scattered field is [11], [27]:

$$\begin{aligned} p_s(\vec{r}_2, t) = \int_{V'}\int_T \Bigg[& \frac{1}{\rho_0}\nabla(\Delta\rho(\vec{r}_1)) \cdot \nabla p_1(\vec{r}_1, t_1) \\ & - \frac{2\Delta c(\vec{r}_1)}{c_0^3}\frac{\partial^2 p_1(\vec{r}_1, t_1)}{\partial t^2}\Bigg] \quad G(\vec{r}_1, t_1 \mid \vec{r}_2, t) dt_1 \, d^3\vec{r}_1 \end{aligned} \tag{2.59}$$

where G is the free space Green's function:

$$G(\vec{r}_1, t_1 \mid \vec{r}_2, t) = \frac{\delta(t - t_1 - \frac{|\vec{r}_2 - \vec{r}_1|}{c_0})}{4\pi \mid \vec{r}_2 - \vec{r}_1 \mid} \tag{2.60}$$

$d^3\vec{r}_1$ means integrating w.r.t. $\vec{r}_1$ over the volume V', and T denotes integration over time.

We denote by

$$F_{op} = \frac{1}{\rho_0}\nabla(\Delta\rho(\vec{r}_1)) \cdot \nabla \quad - \frac{2\Delta c(\vec{r}_1)}{c_0^3}\frac{\partial^2}{\partial t^2} \tag{2.61}$$

the scattering operator.

The pressure field inside the scattering region is:

$$p_1(\vec{r}, t) = p_i(\vec{r}, t) + p_s(\vec{r}, t) \tag{2.62}$$

where p_i is the incident pressure field. As can be seen, the integral can not be solved directly. To solve it we apply the Born-Neumann expansion [30]. If G_i symbolizes the integral operator representing Green's function and the integration and F_{op} the scattering operator, then the first order Born approximation can be written:

$$p_{s_1}(\vec{r}_2, t) = G_i F_{op}\, p_i(\vec{r}_1, t_1) \tag{2.63}$$

Here p_s has been set to zero in (2.62). Inserting p_{s_1} in (2.62) and then in (2.59) we arrive at

$$\begin{aligned} p_{s_2}(\vec{r}_2, t) &= G_i F_{op}\left[p_i(\vec{r}_1, t_1) + G_i F_{op}\, p_i(\vec{r}_1, t_1)\right] \\ &= G_i F_{op}\, p_i(\vec{r}_1, t_1) + \left[G_i F_{op}\right]^2 p_i(\vec{r}_1, t_1) \end{aligned} \tag{2.64}$$

It is emphasized here that G_i indicates an integral over $\vec{r}_1$ and t_1, and not the pressure at point $\vec{r}_1$ and time t_1 but over the volume of V' and time T indicated by $\vec{r}_1$ and t_1.

The general expression for the scattered field then is:

$$\begin{aligned} p_s(\vec{r}_2, t) = \quad & G_i F_{op}\, p_i(\vec{r}_1, t_1) \quad + \\ & \left[G_i F_{op}\right]^2 p_i(\vec{r}_1, t_1) \quad + \\ & \left[G_i F_{op}\right]^3 p_i(\vec{r}_1, t_1) \quad + \\ & \left[G_i F_{op}\right]^4 p_i(\vec{r}_1, t_1) \quad + \cdots \end{aligned} \tag{2.65}$$

Terms involving $\left[G_i F_{op}\right]^N p_i(\vec{r}_1, t_1)$, where $N > 1$, describe multiple scattering of order N. Usually the scattering from small obstacles is considered weak so higher order terms can be neglected. Thus, a useful approximation is to employ only the first term in the expansion. This corresponds to the first order Born-approximation.

Using this (2.59) can be approximated by (note the replacement of $p_1(\vec{r}_1, t_1)$ with $p_i(\vec{r}_1, t_1)$):

$$\begin{aligned} p_s(\vec{r}_2, t) \approx \int_{V'}\int_T & \left[\frac{1}{\rho_0}\nabla(\Delta\rho(\vec{r}_1)) \cdot \nabla p_i(\vec{r}_1, t_1) \right. \\ & \left. - \frac{2\Delta c(\vec{r}_1)}{c_0^3}\frac{\partial^2 p_i(\vec{r}_1, t_1)}{\partial t^2}\right] \quad G(\vec{r}_1, t_1 \mid \vec{r}_2, t) dt_1\, d^3\vec{r}_1 \end{aligned} \tag{2.66}$$

So in order to calculate the scattered field, the incident field for the homogeneous medium must be calculated.

2.6.3 Calculation of the incident field

The incident field is generated by the ultrasound transducer assuming no other sources exist in the tissue. The field is conveniently calculated by employing the velocity potential $\psi(\vec{r}, t)$, and enforcing appropriate boundary conditions [31], [32]. The velocity potential satisfies the following wave equation for the homogeneous medium:

$$\nabla^2\psi - \frac{1}{{c_0}^2}\frac{\partial^2\psi}{\partial^2 t} = 0 \tag{2.67}$$

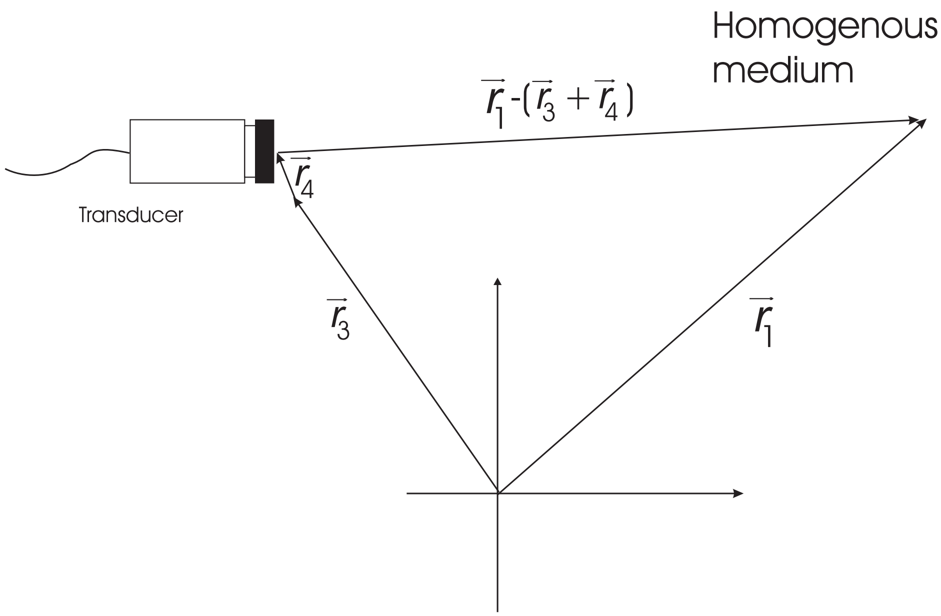


Figure 2.17: Coordinate system for calculating the incident field.

and the pressure is calculated from:

$$p(\vec{r},t) = \rho_0 \frac{\partial \psi(\vec{r},t)}{\partial t} \tag{2.68}$$

The coordinate system shown in Fig. 2.17 is used in the calculation. The particle velocity normal to the transducer surface is denoted by $v(\vec{r}_3 + \vec{r}_4, t)$, where $\vec{r}_3$ identifies the position of the transducer and $\vec{r}_4$ a point on the transducer surface relative to $\vec{r}_3$.

The solution to the homogeneous wave equation is [32]:

$$\psi(\vec{r}_1 + \vec{r}_3, t) = \int_S \int_T v(\vec{r}_3 + \vec{r}_4, t_3) g(\vec{r}_1, t \mid \vec{r}_3 + \vec{r}_4, t_3) dt_3 \, d^2\vec{r}_4 \tag{2.69}$$

when the transducer is mounted in a rigid infinite planar baffle. S denotes the transducer surface.

g is the Green's function for a bounded medium and is

$$g(\vec{r}_1, t \mid \vec{r}_3 + \vec{r}_4, t_3) = \frac{\delta(t - t_3 - \frac{|\vec{r}_1 - \vec{r}_3 - \vec{r}_4|}{c_0})}{2\pi \mid \vec{r}_1 - \vec{r}_3 - \vec{r}_4 \mid} \tag{2.70}$$

$\mid \vec{r}_1 - \vec{r}_3 - \vec{r}_4 \mid$ is the distance from S to the point where the field is calculated and c_0 the mean propagation velocity. The field is calculated under the assumption of radiation into an isotropic, homogeneous, non-dissipative medium.

If a slightly curved transducer is used, an additional term is introduced as shown in Morse & Feshbach [33]. This term is called the second order diffraction term in Penttinen & Luukkala [18]. It can be shown to vanish for a planar transducer, and as long as the transducer is only slightly curved and large compared to the wavelength of the ultrasound, the resulting expression is a good approximation to the pressure field [18].

If the particle velocity is assumed to be uniform over the surface of the transducer, (2.69) can be reduced to [34]:

$$\psi(\vec{r}_1, \vec{r}_3, t) = \int_T v(t_3) \int_S g(\vec{r}_1, t \mid \vec{r}_3 + \vec{r}_4, t_3) d^2\vec{r}_4 \, dt_3 \tag{2.71}$$

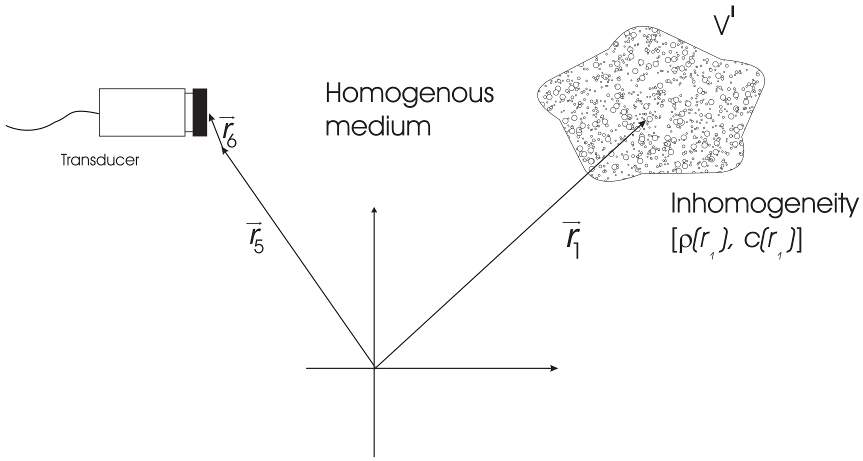


Figure 2.18: Coordinate system for calculating the received signal.

This is the spatial impulse response previously derived, and the sound pressure for the incident field then is:

$$p(\vec{r}_1,\vec{r}_3,t) = \rho_0 \frac{\partial \psi(\vec{r}_1,\vec{r}_3,t)}{\partial t} = \rho_0 v(t) \underset{t}{\star} \frac{\partial h(\vec{r}_1,\vec{r}_3,t)}{\partial t} \tag{2.72}$$

or

$$p(\vec{r}_1,\vec{r}_3,t) = \rho_0 \frac{\partial v(t)}{\partial t} \underset{t}{\star} h(\vec{r}_1,\vec{r}_3,t) \tag{2.73}$$

2.6.4 Calculation of the received signal

The received signal is the scattered pressure field integrated over the transducer surface, convolved with the electro-mechanical impulse response, $E_m(t)$, of the transducer. To calculate this we introduce the coordinate system shown in Fig. 2.18. $\vec{r}_6 + \vec{r}_5$ indicates a receiving element on the surface of the transducer that is located at $\vec{r}_5$. The received signal is:

$$p_r(\vec{r}_5,t) = E_m(t) \underset{t}{\star} \int_S p_s(\vec{r}_6+\vec{r}_5,t) d^2\vec{r}_6 \tag{2.74}$$

The scattered field is:

$$p_s(\vec{r}_6+\vec{r}_5,t) = \tfrac{1}{2} \int_{V'} \int_T F_{op}\left[p_i(\vec{r}_1,t_1)\right] \frac{\delta(t-t_1-\frac{|\vec{r}_6+\vec{r}_5-\vec{r}_1|}{c_0})}{2\pi \mid \vec{r}_6+\vec{r}_5-\vec{r}_1 \mid} dt_1 \, d^3\vec{r}_1 \tag{2.75}$$

Combining this with (2.74) and comparing with (2.9) we see that p_r includes Green's function for bounded space integrated over the transducer surface, which is equal to the spatial impulse response. Inserting the expression for p_i and performing the integration over the transducer surface and over time, results in:

$$p_r(\vec{r}_5,t) = E_m(t) \underset{t}{\star} \tfrac{1}{2} \int_{V'} F_{op}\left[\rho_0 \frac{\partial v(t)}{\partial t} \underset{t}{\star} h(\vec{r}_1,\vec{r}_3,t)\right] \underset{t}{\star} h(\vec{r}_5,\vec{r}_1,t) d^3\vec{r}_1 \tag{2.76}$$

If the position of the transmitting and the receiving transducer is the same ($\vec{r}_3 = \vec{r}_5$), then a simple rearrangement of (2.76) yields:

$$p_r(\vec{r}_5,t) = \frac{\rho_0}{2} E_m(t) \underset{t}{\star} \frac{\partial v(t)}{\partial t} \underset{t}{\star} \int_{V'} F_{op}\left[h_{pe}(\vec{r}_1,\vec{r}_5,t)\right] d^3\vec{r}_1 \tag{2.77}$$

where

$$h_{pe}(\vec{r}_1, \vec{r}_5, t) = h(\vec{r}_1, \vec{r}_5, t) \underset{t}{\star} h(\vec{r}_5, \vec{r}_1, t) \tag{2.78}$$

is the pulse-echo spatial impulse response.

The calculated signal is the response measured for one given position of the transducer. For a B-mode scan picture a number of scan-lines is measured and combined to a picture. To analyze this situation, the last factor in (2.77) is explicitly written out

$$\int_{V'} \left[\frac{1}{\rho_0} \nabla(\Delta\rho(\vec{r}_1)) \cdot \nabla h_{pe}(\vec{r}_1, \vec{r}_5, t) - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 h_{pe}(\vec{r}_1, \vec{r}_5, t)}{\partial t^2} \right] d^3\vec{r}_1 \tag{2.79}$$

From section 2.6.3 it is know that H_{pe} is a function of the distance between $\vec{r}_1$ and $\vec{r}_5$, while $\Delta\rho$ and Δc only are functions of $\vec{r}_1$. So when $\vec{r}_5$ is varied over the volume of interest, the resulting image is a spatial non-stationary convolution between $\Delta\rho$, Δc and a modified form of the pulse-echo spatial impulse response.

If we assume that the pulse-echo spatial impulse is slowly varying so that the spatial frequency content is constant over a finite volume, then (2.79) can be rewritten

$$\int_{V'} \left[\frac{1}{\rho_0} \Delta\rho(\vec{r}_1) \nabla^2 h_{pe}(\vec{r}_1, \vec{r}_5, t) - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 h_{pe}(\vec{r}_1, \vec{r}_5, t)}{\partial t^2} \right] d^3\vec{r}_1 \tag{2.80}$$

h_{pe} is a function of the distance between the transducer and the scatterer or equivalently of the corresponding time given by

$$t = \frac{\mid \vec{r}_1 - \vec{r}_5 \mid}{c_0} \tag{2.81}$$

The Laplace operator is the second derivative w.r.t. the distance, which can be approximated with the second derivative w.r.t. time. So

$$\nabla^2 h_{pe}(\vec{r}_1, \vec{r}_5, t) = \frac{1}{c_0^2} \frac{\partial^2 h_{pe}(\vec{r}_1, \vec{r}_5, t)}{\partial t^2} \tag{2.82}$$

assuming only small deviations from the mean propagation velocity.

Using these approximations, (2.77) can be rewritten:

$$p_r(\vec{r}_5, t) = \frac{\rho_0}{2c_0^2} E_m(t) \underset{t}{\star} \frac{\partial v^3(t)}{\partial t^3} \underset{t}{\star} \int_{V'} \left[\frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0} \right] h_{pe}(\vec{r}_1, \vec{r}_5, t) d^3\vec{r}_1 \tag{2.83}$$

Symbolically this is written

$$p_r(\vec{r}_5, t) = v_{pe}(t) \underset{t}{\star} f_m(\vec{r}_1) \underset{r}{\star} h_{pe}(\vec{r}_1, \vec{r}_5, t) \tag{2.84}$$

$\underset{r}{\star}$ denotes spatial convolution. v_{pe} is the pulse-echo wavelet which includes the transducer excitation and the electro-mechanical impulse response during emission and reception of the pulse. f_m accounts for the inhomogeneities in the tissue due to density and propagation velocity perturbations which give rise to the scattered signal. h_{pe} is the modified pulse-echo spatial impulse response that relates the transducer geometry to the spatial extent of the scattered field. Explicitly written out these terms are:

$$v_{pe}(t) = \frac{\rho_0}{2c_0^2} E_m(t) \underset{t}{\star} \frac{\partial v^3(t)}{\partial t^3} \tag{2.85}$$

$$f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0} \tag{2.86}$$

$$h_{pe}(\vec{r}_1, \vec{r}_5, t) = h(\vec{r}_1, \vec{r}_5, t) \underset{t}{\star} h(\vec{r}_5, \vec{r}_1, t) \tag{2.87}$$

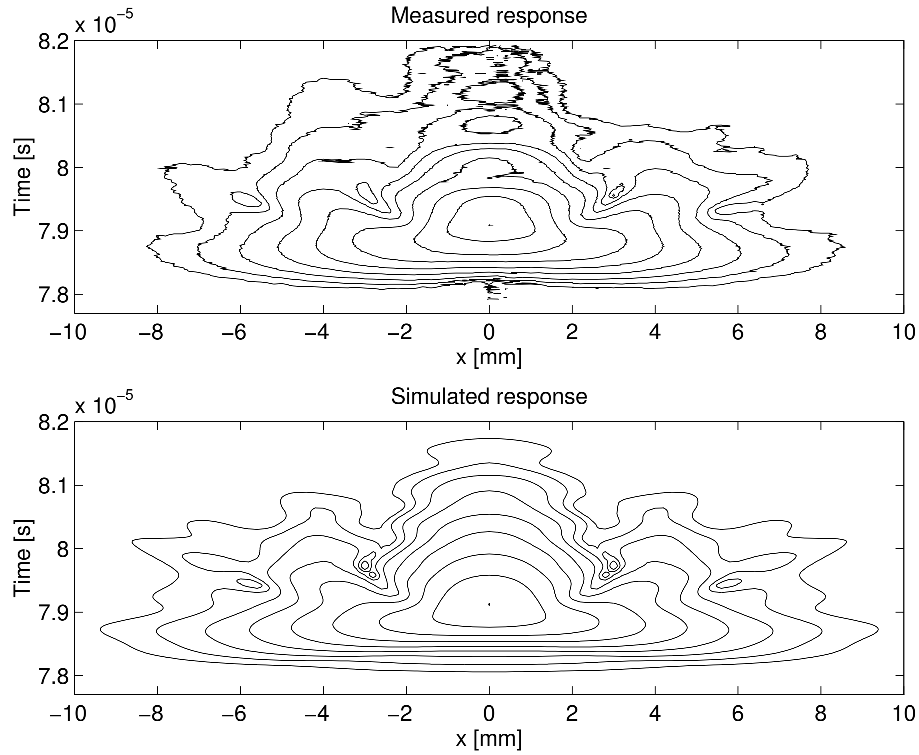


Figure 2.19: Measured and simulated pulse-echo response for a concave transducer. The axial distance was 60 mm and the small scatterer was moved in the lateral direction. The envelope of the RF signal is shown as 6 dB contours.

Expression (2.84) consists of three distinct terms. The interesting signal, and the one that should be displayed in medical ultrasound, is $f_m(\vec{r}_1)$. We, however, measure a time and spatially smoothed version of this, which obscures the finer details in the picture. The smoothing consists of a convolution in time with a fixed wavelet $v_{pe}(t)$ and a spatial convolution with a spatially varying $h_{pe}(\vec{r}_1, \vec{r}_5, t)$.

2.6.5 Example of pulse echo response

To show that the pulse-echo response can be calculated to good accuracy, a single example is shown in Fig. 2.19 for a concave transducer (r = 8.1 mm) with a focus at R = 150 mm. The measured and simulated responses were obtained at a distance of 60 mm from the transducer surface. The measured pressure field was acquired by moving a needle pointing toward the transducer in steps of 0.2 mm in the lateral direction, making measurements in a plane containing the acoustical axis of the transducer. The simulated field was calculated by measuring v_{pe} as the response from a planar reflector, and then using (2.32) and (2.84) to calculate the field. The envelope of the RF-signals is shown as a contour plot with 6 dB between the contours. The plots span 20 mm in the lateral direction and 4 μs in the axial direction.

2.6.6 Attenuation effects

The model includes attenuation of the pulse due to propagation, but not the dispersive attenuation of the wave observed when propagating in tissue. This changes the pulse continuously as it propagates down through the tissue. Not including dispersive attenuation is, however, not a serious drawback of the theory, as this change of the pulse can be lumped into the already spatially varying h_{pe}. Or, if in the far field and assuming a homogeneous, dispersive attenuation, then an attenuation transfer function can be convolved onto v_{pe} to yield an attenuated pulse.

Submerging the transducer into a *homogeneously* attenuating medium will modify the propagation of the spherical waves, which will change continuously as a function of distance from the transducer. The spatial impulse is then changed to

$$h_{att}(t, \vec{r}) = \int_T \int_S a(t-\tau, |\vec{r}+\vec{r}_1|) \frac{\delta(\tau - \frac{|\vec{r}+\vec{r}_1|}{c})}{|\vec{r}+\vec{r}_1|} dS d\tau \tag{2.88}$$

when linear propagation is assumed and the attenuation is the same throughout the medium. a is the attenuation impulse response. The spherical wave is convolved with the distance dependent attenuation impulse response and spherical waves emanating from different parts of the aperture are convolved with different attenuation impulse responses.

A model for the attenuation must be introduced in order to solve the integral. Ultrasound propagating in tissue experiences a nearly linear with frequency attenuation and a commonly used attenuation amplitude transfer function is

$$|A'(f, |\vec{r}|)| = \exp(-\beta' f |\vec{r}|) \tag{2.89}$$

where β' is attenuation in nepers per meter. We here prefer to split the attenuation into a frequency dependent and a frequency independent part term as

$$|A(f, |\vec{r}|)| = \exp(-\alpha|\vec{r}|)\exp(-\beta(f-f_0)|\vec{r}|) \tag{2.90}$$

α is the frequency independent attenuation coefficient, and f_0 the transducer center frequency. The phase of the attenuation need also be considered. Kak and Dines [35] introduced a linear with frequency phase response

$$\Theta(f) = 2\pi f \tau_b |\vec{r}| \tag{2.91}$$

where τ_b is the bulk propagation delay per unit length and is equal to $1/c$. This, however, results in an attenuation impulse response that is non-causal. Gurumurthy and Arthur [36] therefore suggested using a minimum phase impulse response, where the amplitude and phase spectrum form a Hilbert transform pair. The attenuation spectrum is then given by

$$\begin{aligned} A(f, |\vec{r}|) &= \exp(-\alpha|\vec{r}|)\exp(-\beta(f-f_0)|\vec{r}|) \\ &\times \exp(-j2\pi f(\tau_b + \tau_m \frac{\beta}{\pi^2})|\vec{r}|) \\ &\times \exp(j\frac{2f}{\pi}\beta|\vec{r}|\ln(2\pi f)) \end{aligned} \tag{2.92}$$

where τ_m is the minimum phase delay factor. Gurumurthy and Arthur [36] suggest a τ_m value of 20 in order to fit the dispersion found in tissue.

The inverse Fourier transform of (2.92) must be inserted into (2.88) and the integral has to be solved for the particular transducer geometry. This is clearly a difficult, if not impossible, task and some approximations must be introduced. All spherical waves arrive at nearly the same time instance, if the distance to the field point is much larger than the transducer aperture. In this case the attenuation function is, thus, the same for all waves and the result is a convolution between the attenuation impulse response and the spatial impulse response, which is a Dirac impulse. A spatial impulse response other than a Dirac function indicates that the spherical waves arrive at different times. Multiplying the arrival time with the propagation velocity gives the distance to the points on the aperture contributing to the response at that time instance. A first approximation is, therefore, to multiply the non-attenuated spatial impulse response with the proper frequency independent term. This approximation also assumes that the span of values for $|\vec{r}+\vec{r_1}|$ is so small that both $1/|\vec{r}+\vec{r}_1|$ and the attenuation can be assumed to be roughly constant.

The frequency dependent function will also change for the different values of the spatial impulse response. A non-stationary convolution must, thus, be performed. One possible method to avoid this is to assume that the frequency dependent attenuation impulse response is constant for the time and, thus, distances $|\vec{r}+\vec{r}_1|$ where h is non-zero. The mean distance is then used in (2.92) and the inverse Fourier transform of $A(f, |\vec{r}_{mid}|)$ is convolved with $h(t, \vec{r})$. The accuracy of the approach depends on the duration of h and of the attenuation. The error in dB for a concave transducer

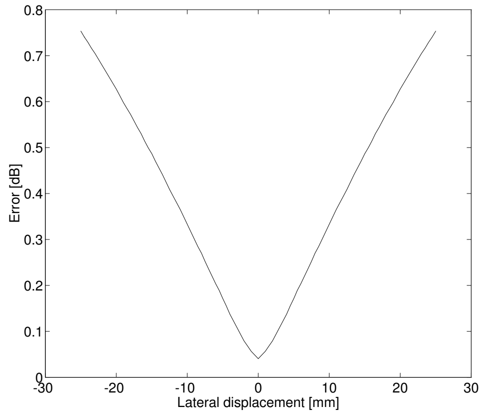


Figure 2.20: Error of assuming a non-varying frequency dependent attenuation over the duration of the spatial impulse response.

with a radius of 10 mm focused at 100 mm and an attenuation of 0.5 dB/[MHz cm] is shown in Fig. 2.20. The axial distance to the transducer is 50 mm.

An example of the influence of attenuation on the point spread function (PSF) is shown in Fig. 2.21. A concave transducer with a radius of 8 mm, center frequency of 3 MHz, and focused at 100 mm was used. Fig. 2.21 shows point spread functions calculated under different conditions. The logarithmic envelope of the received pressure is displayed as a function of time and lateral displacement. The left most graph shows the normalized PSF for the transducer submerged in a non-attenuating medium. The distance to the field point is 60 mm and the function is shown for lateral displacements from -8 to 8 mm. Introducing a 0.5 dB/[MHz cm] attenuation yields the normalized PSF shown in the middle. The central core of the PSF does not change significantly, but the shape at -30 dB and below are somewhat different from the non-attenuated response. A slightly broader and longer function is seen, but the overall shape is the same.

An commonly used approach to characterize the field is to include the attenuation into the basic one-dimensional pulse, and then use the non-attenuated spatial impulse response in calculating the PSF. This is the approach used in the rightmost graph in Fig. 2.21. All attenuation is included in the pulse and the spatial impulse response calculated in the leftmost graph is used for making the PSF. The similarity to the center graph is striking. Apart from a slightly longer response, nearly all features of the field are the same. It is, thus, appropriate to estimate the attenuated one-dimensional pulse and reconstruct the whole field from this and knowledge of the transducer geometry.

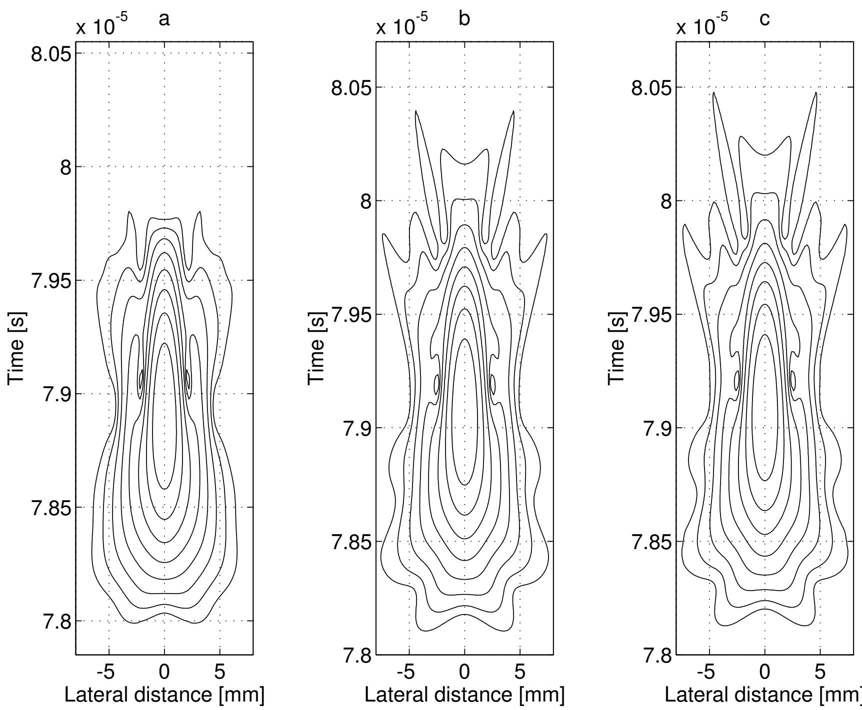


Figure 2.21: Contour plots of point spread functions for different media and calculation methods. a: Non attenuating medium. b: 0.5 dB/[MHz cm] attenuation. c: 0.5 dB/[MHz cm] attenuation on the one-dimensional pulse. There is 6 dB between the contour lines. The distance to the transducer is 60 mm.

CHAPTER

THREE

Ultrasound imaging

Modern medical ultrasound scanners are used for imaging nearly all soft tissue structures in the body. The anatomy can be studied from gray-scale B-mode images, where the reflectivity and scattering strength of the tissues are displayed. The imaging is performed in real time with 20 to 100 images per second. The technique is widely used since it does not use ionizing radiation and is safe and painless for the patient.

This chapter gives a short introduction to modern ultrasound imaging using array transducers. Part of the chapter is based on [9] and [37].

3.1 Fourier relation

This section derives a simple relation between the oscillation of the transducer surface and the ultrasound field. It is shown that field in the far-field can be found by a simple one-dimensional Fourier transform of the one-dimensional aperture pattern. This might seem far from the actual imaging situation in the near field using pulsed excitation, but the approach is very convenient in introducing all the major concepts like main and side lobes, grating lobes, etc. It also very clearly reveals information about the relation between aperture properties and field properties.

3.1.1 Derivation of Fourier relation

Consider a simple line source as shown in Fig. 3.1 with a harmonic particle speed of $U_0 \exp(j\omega t)$. Here U_0 is the vibration amplitude and ω is its angular frequency. The line element of length dx generates an increment in pressure of [13]

$$dp = j\frac{\rho_0 ck}{4\pi r'}U_0 a_p(x) e^{j(\omega t - kr')}dx, \tag{3.1}$$

where ρ_0 is density, c is speed of sound, $k = \omega/c$ is the wavenumber, and $a_p(x)$ is an amplitude scaling of the individual parts of the aperture. In the far-field ($r \ll L$) the distance from the radiator to the field points is (see Fig. 3.1)

$$r' = r - x\sin\theta \tag{3.2}$$

The emitted pressure is found by integrating over all the small elements of the aperture

$$p(r,\theta,t) = j\frac{\rho_0 c U_0 k}{4\pi}\int_{-\infty}^{+\infty} a_p(x)\frac{e^{j(\omega t - r')}}{r'}dx. \tag{3.3}$$

Notice that $a_p(x) = 0$ if $|x| > L/2$. Here r' can be replaced with r, if the extent of the array is small compared to the distance to the field point ($r \ll L$). Using this approximation and inserting (3.2) in (3.3) gives

$$p(r,\theta,t) = j\frac{\rho_0 c U_0 k}{4\pi r}\int_{-\infty}^{+\infty} a_p(x) e^{j(\omega t - kr + kx\sin\theta)}dx = \frac{\rho_0 c U_0 k}{4\pi r}e^{j(\omega t - kr)}\int_{-\infty}^{+\infty} a_p(x) e^{jkx\sin\theta}dx, \tag{3.4}$$

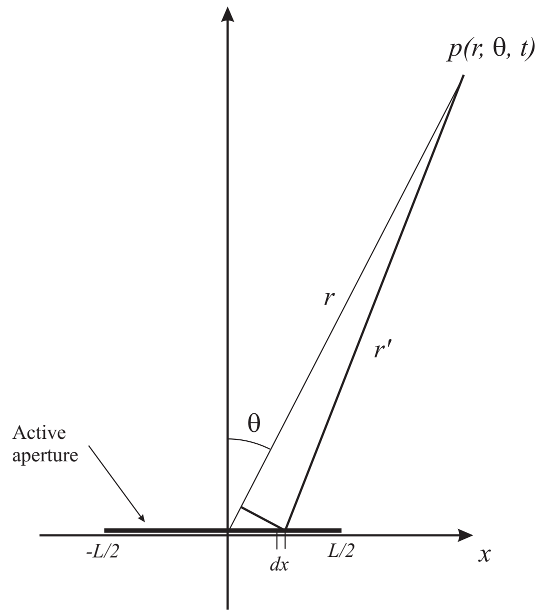


Figure 3.1: Geometry for line aperture.

since ωt and kr are independent of x. Hereby the pressure amplitude of the field for a given frequency can be split into two factors:

$$\begin{aligned} P_{ax}(r) &= \frac{\rho_0 c U_0 k L}{4\pi r} \\ H(\theta) &= \frac{1}{L}\int_{-\infty}^{+\infty} a_p(x) e^{jkx\sin\theta} dx \\ P(r,\theta) &= P_{ax}(r) H(\theta) \end{aligned} \tag{3.5}$$

The first factor $P_{ax}(r)$ characterizes how the field drops off in the axial direction as a factor of distance, and $H(\theta)$ gives the variation of the field as a function of angle. The first term drops off with $1/r$ as for a simple point source and $H(\theta)$ is found from the aperture function $a_p(x)$. A slight rearrangement gives[1]

$$H(\theta) = \frac{1}{L}\int_{-\infty}^{+\infty} a_p(x) e^{j2\pi x f \frac{\sin\theta}{c}} dx = \frac{1}{L}\int_{-\infty}^{+\infty} a_p(x) e^{j2\pi x f'} dx. \tag{3.6}$$

This very closely resembles the standard Fourier integral given by

$$\begin{aligned} G(f) &= \int_{-\infty}^{+\infty} g(t) e^{-j2\pi t f} dt \\ g(t) &= \int_{-\infty}^{+\infty} G(f) e^{j2\pi t f} df \end{aligned} \tag{3.7}$$

There is, thus, a Fourier relation between the radial beam pattern and the aperture function, and the normal Fourier relations can be used for understanding the beam patterns for typical apertures.

[1]The term $1/L$ is included to make $H(\theta)$ a unit less number.

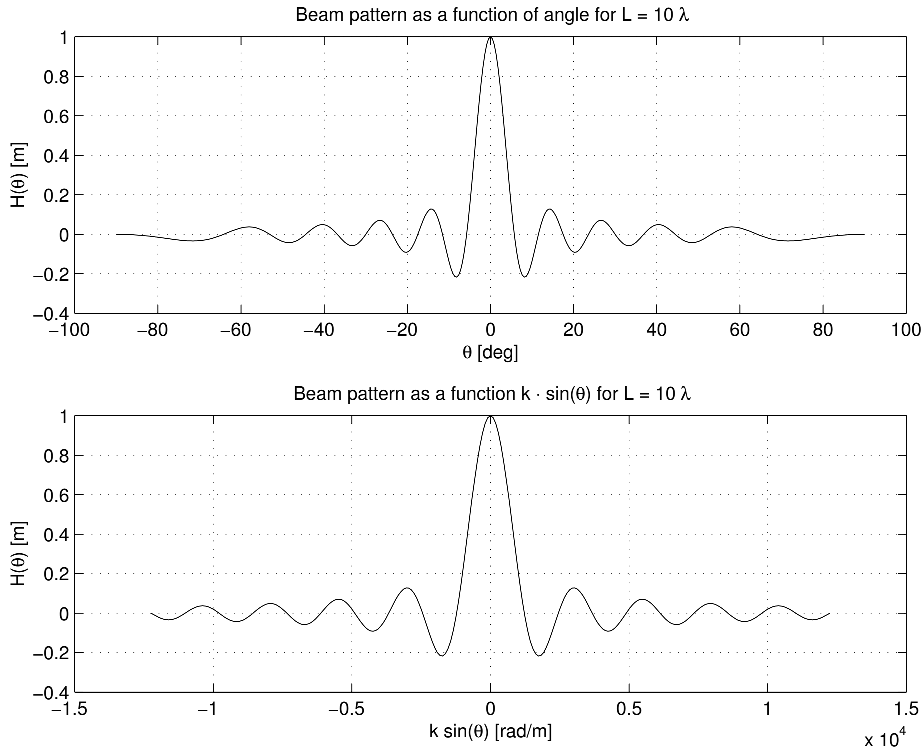


Figure 3.2: Angular beam pattern for a line aperture with a uniform aperture function as a function of angle (top) and as a function of $k\sin(\theta)$ (bottom).

3.1.2 Beam patterns

The first example is for a simple line source, where the aperture function is constant such that

$$a_p(x) = \begin{cases} 1 & |x| \leq L/2 \\ 0 & \text{else} \end{cases} \tag{3.8}$$

The angular factor is then

$$H(\theta) = \frac{\sin(\pi L f \frac{\sin\theta}{c})}{\pi L f \frac{\sin\theta}{c}} = \frac{\sin(\frac{k}{2} L \sin\theta)}{\frac{k}{2} L \sin\theta} \tag{3.9}$$

A plot of the sinc function is shown in Fig. 3.2. A single main lobe can be seen with a number of side lobe peaks. The peaks fall off proportionally to k or f. The angle of the first zero in the function is found at

$$\sin\theta = \frac{c}{Lf} = \frac{\lambda}{L}. \tag{3.10}$$

The angle is, thus, dependent on the frequency and the size of the array. A large array or a high emitted frequency, therefore, gives a narrow main lobe.

The magnitude of the first sidelobe relative to the mainlobe is given by

$$\frac{H(\arcsin(\frac{3c}{2Lf}))}{H(0)} = L\frac{\sin(3\pi/2)}{3\pi/2}/L = \frac{2}{3\pi} \tag{3.11}$$

The relative sidelobe level is, thus, independent of the size of the array and of the frequency, and is solely determined by the aperture function $a_p(x)$ through the Fourier relation. The large discontinuities of $a_p(x)$, thus, give rise to the high side lobe level, and they can be reduced by selecting an aperture function that is smoother like a Hanning window or a Gaussian shape.

Modern ultrasound transducers consist of a number of elements each radiating ultrasound energy. Neglecting the phasing of the element (see Section 3.2) due to the far-field assumption, the aperture function can be described by

$$a_p(x) = a_{ps}(x) * \sum_{n=-N/2}^{N/2} \delta(x - d_x n), \tag{3.12}$$

where $a_{ps}(x)$ is the aperture function for the individual elements, d_x is the spacing (pitch) between the centers of the individual elements, and N is the number of elements in the array. Using the Fourier relationship the angular beam pattern can be described by

$$H_p(\theta) = H_{ps}(\theta) H_{per}(\theta), \tag{3.13}$$

where

$$\sum_{n=-N/2}^{N/2} \delta(x - d_x n) \leftrightarrow H_{per}(\theta) = \sum_{n=-N/2}^{N/2} e^{-jnd_x ksin\theta}. = \sum_{n=-N/2}^{N/2} e^{-j2\pi \frac{fsin\theta}{c} nd_x}. \tag{3.14}$$

Summing the geometric series gives

$$H_{per}(\theta) = \frac{\sin\left((N+1)\frac{k}{2} d_x \sin\theta\right)}{\sin\left(\frac{k}{2} d_x \sin\theta\right)} \tag{3.15}$$

is the Fourier transform of series of delta functions. This function repeats itself with a period that is a multiple of

$$\begin{aligned} \pi &= \frac{k}{2} d_x \sin\theta \\ \sin\theta &= \frac{\pi}{k d_x} = \frac{\lambda}{d_x}. \end{aligned} \tag{3.16}$$

This repetitive function gives rise to the grating lobes in the field. An example is shown in Fig. 3.3.

The grating lobes are due to the periodic nature of the array, and corresponds to sampling of a continuous time signal. The grating lobes will be outside a ± 90 deg. imaging area if

$$\begin{aligned} \frac{\lambda}{d_x} &= 1 \\ d_x &= \lambda \end{aligned} \tag{3.17}$$

Often the beam is steered in a direction and in order to ensure that grating lobes do not appear in the image, the spacing or pitch of the elements is selected to be $d_x = \lambda/2$. This also includes ample margin for the modern transducers that often have a very broad bandwidth.

An array beam can be steered in a direction by applying a time delay on the individual elements. The difference in arrival time for a given direction θ_0 is

$$\tau = \frac{d_x \sin\theta_0}{c} \tag{3.18}$$

Steering in a direction θ_0 can, therefore, be accomplished by using

$$\sin\theta_0 = \frac{c\tau}{d_x} \tag{3.19}$$

where τ is the delay to apply to the signal on the element closest to the center of the array. A delay of 2τ is then applied on the second element and so forth. The beam pattern for the grating lobe is then replaced by

$$H_{per}(\theta) = \frac{\sin\left((N+1)\frac{k}{2} d_x \sin\left(\theta - \frac{c\tau}{d_x}\right)\right)}{\sin\left(\frac{k}{2} d_x \sin\left(\theta - \frac{c\tau}{d_x}\right)\right)}. \tag{3.20}$$

Notice that the delay is independent of frequency, since it is essentially only determined by the speed of sound.

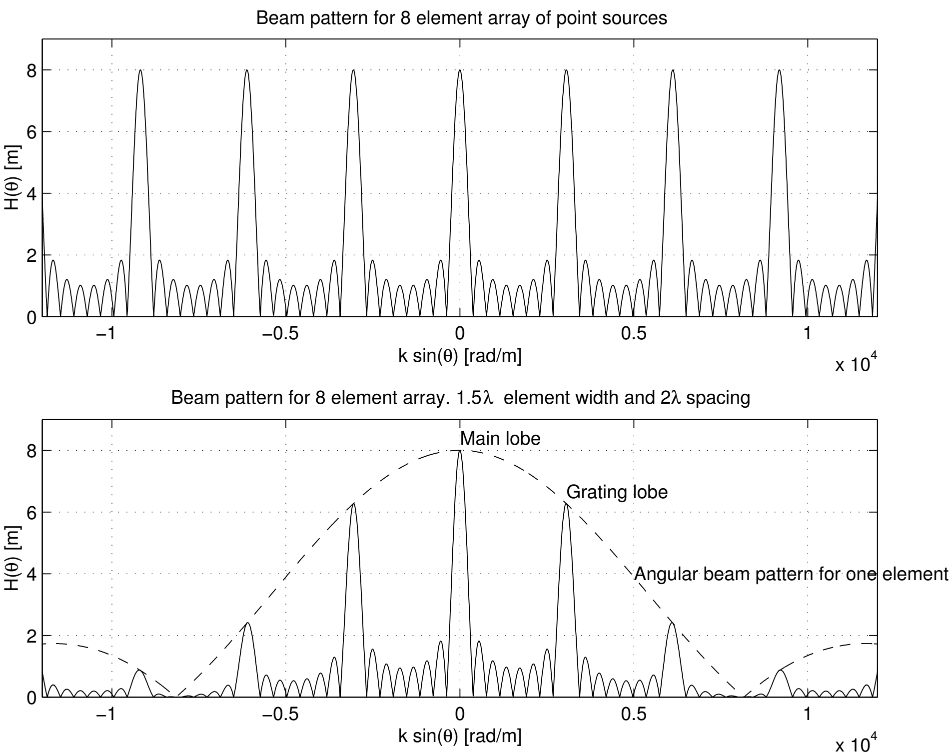


Figure 3.3: Grating lobes for array transducer consisting of 8 point elements (top) and of 8 elements with a size of 1.5λ (bottom). The pitch (or distance between the elements) is 2λ.

3.2 Focusing

The essence of focusing an ultrasound beam is to align the pressure fields from all parts of the aperture to arrive at the field point at the same time. This can be done through a physically curved aperture, through a lens in front of the aperture, or by the use of electronic delays for multi-element arrays. All seek to align the arrival of the waves at a given point through delaying or advancing the fields from the individual elements. The delay (positive or negative) is determined using ray acoustics. The path length from the aperture to the point gives the propagation time and this is adjusted relative to some reference point. The propagation from the center of the aperture element to the field point is

$$t_i = \frac{1}{c}\sqrt{(x_i - x_f)^2 + (y_i - y_f)^2 + (z_i - z_f)^2} \tag{3.21}$$

where (x_f, y_f, z_f) is the position of the focal point, (x_i, y_i, z_i) is the center for the physical element number i, c is the speed of sound, and t_i is the calculated propagation time.

A point is selected on the whole aperture as a reference for the imaging process. The propagation time for this is

$$t_c = \frac{1}{c}\sqrt{(x_c - x_f)^2 + (y_c - y_f)^2 + (z_c - z_f)^2} \tag{3.22}$$

where (x_c, y_c, z_c) is the reference center point on the aperture. The delay to use on each element of the array is then

$$\Delta t_i = \frac{1}{c}\left(\sqrt{(x_c - x_f)^2 + (y_c - y_f)^2 + (z_c - z_f)^2} - \sqrt{(x_i - x_f)^2 + (y_i - y_f)^2 + (z_i - z_f)^2}\right) \tag{3.23}$$

Notice that there is no limit on the selection of the different points, and the beam can, thus, be steered in a preferred direction.

The arguments here have been given for emission from an array, but they are equally valid during reception of the ultrasound waves due to acoustic reciprocity. At reception it is also possible to change the focus as a function of time and thereby obtain a dynamic tracking focus. This is used by all modern ultrasound scanners, Beamformers based on analog technology makes it possible to create several receive foci and the newer digital scanners change the focusing continuously for every depth in receive. A single focus is only possible in transmit and composite imaging is therefore often used in modern imaging. Here several pulse emissions with focusing at different depths in the same direction are used and the received signals are combined to form one image focused in both transmit and receive at different depths (composit imaging).

The focusing can, thus, be defined through time lines as:

From time	Focus at
0	x_1, y_1, z_1
t_1	x_1, y_1, z_1
t_2	x_2, y_2, z_2
$\vdots$	$\vdots$

For each focal zone there is an associated focal point and the time from which this focus is used. The arrival time from the field point to the physical transducer element is used for deciding which focus is used. Another possibility is to set the focusing to be dynamic, so that the focus is changed as a function of time and thereby depth. The focusing is then set as a direction defined by two angles and a starting point on the aperture.

Section 3.1 showed that the side and grating lobes of the array can be reduced by employing apodization of the elements. Again a fixed function can be used in transmit and a dynamic function in receive defined by

From time	Apodize with
0	$a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$
t_1	$a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$
t_2	$a_{2,1}, a_{2,2}, \cdots a_{2,N_e}$
t_3	$a_{3,1}, a_{3,2}, \cdots a_{3,N_e}$
$\vdots$	$\vdots$

Here $a_{1,1}$ is the amplitude scaling value multiplied onto element 1 after time instance t_1. Typically a Hamming or Gaussian shaped function is used for the apodization. In receive the width of the function is often increased to compensate for attenuation effects and for keeping the point spread function roughly constant. The F-number defined by

$$F = \frac{D}{L} \tag{3.24}$$

where L is the total width of the active aperture and D is the distance to the focus, is often kept constant. More of the aperture is often used for larger depths and a compensation for the attenuation is thereby partly made. An example of the use of dynamic apodization is given in Section 3.6.

3.3 Fields from array transducers

Most modern scanners use arrays for generating and receiving the ultrasound fields. These fields are quite simple to calculate, when the spatial impulse response for a single element is known. This is the approach used in the Field II program, and this section will extend the spatial impulse response to multi element transducers and will elaborate on some of the features derived for the fields in Section 3.1.

Since the ultrasound propagation is assumed to be linear, the individual spatial impulse responses can simply be added. If $h_e(\vec{r}_p, t)$ denotes the spatial impulse response for the element at position $\vec{r}_i$ and the field point $\vec{r}_p$, then the spatial impulse response for the array is

$$h_a(\vec{r}_p, t) = \sum_{i=0}^{N-1} h_e(\vec{r}_i, \vec{r}_p, t), \tag{3.25}$$

assuming all N elements to be identical.

Let us assume that the elements are very small and the field point is far away from the array, so h_e is a Dirac function. Then

$$h_a(\vec{r}_p, t) = \frac{k}{R_p} \sum_{i=0}^{N-1} \delta(t - \frac{|\vec{r}_i - \vec{r}_p|}{c}) \tag{3.26}$$

when $R_p = |\vec{r}_a - \vec{r}_p|$, k is a constant of proportionality, and $\vec{r}_a$ is the position of the array. Thus, h_a is a train of Dirac pulses. If the spacing between the elements is D, then

$$h_a(\vec{r}_p, t) = \frac{k}{R_p} \sum_{i=0}^{N-1} \delta\left(t - \frac{|\vec{r}_a + iD\vec{r}_e - \vec{r}_p|}{c}\right), \tag{3.27}$$

where $\vec{r}_e$ is a unit vector pointing in the direction along the elements. The geometry is shown in Fig. 3.4.

The difference in arrival time between elements far from the transducer is

$$\Delta t = \frac{D \sin \Theta}{c}. \tag{3.28}$$

The spatial impulse response is, thus, a series of Dirac pulses separated by Δt.

$$h_a(\vec{r}_p, t) \approx \frac{k}{R_p} \sum_{i=0}^{N-1} \delta\left(t - \frac{R_p}{c} - i\Delta t\right). \tag{3.29}$$

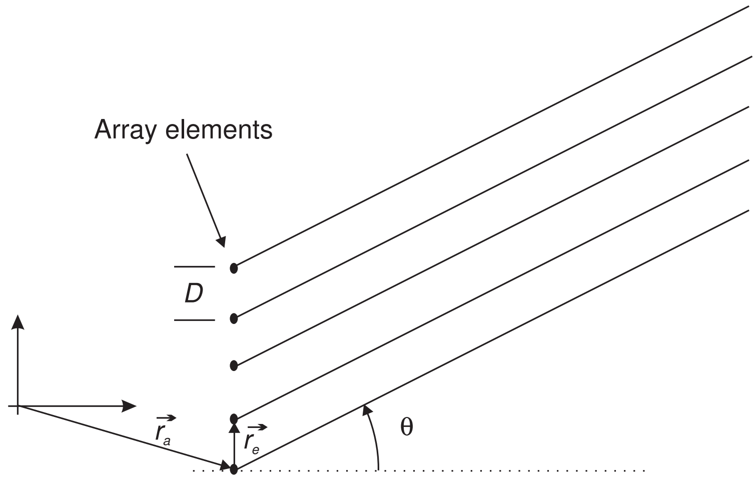


Figure 3.4: Geometry of linear array (from [9], Copyright Cambridge University Press).

The time between the Dirac pulses and the shape of the excitation determines whether signals from individual elements add or cancel out. If the separation in arrival times corresponds to exactly one or more periods of a sine wave, then they are in phase and add constructively. Thus, peaks in the response are found for

$$n\frac{1}{f} = \frac{D\sin\Theta}{c}. \tag{3.30}$$

The main lobe is found for $\Theta = 0$ and the next maximum in the response is found for

$$\Theta = \arcsin\left(\frac{c}{fD}\right) = \arcsin\left(\frac{\lambda}{D}\right). \tag{3.31}$$

For a 3 MHz array with an element spacing of 1 mm, this amounts to $\Theta = 31^\circ$, which will be within the image plane. The received response is, thus, affected by scatterers positioned 31° off the image axis, and they will appear in the lines acquired as grating lobes. The first grating lobe can be moved outside the image plane, if the elements are separated by less than a wavelength. Usually, half a wavelength separation is desirable, as this gives some margin for a broad-band pulse and beam steering.

The beam pattern as a function of angle for a particular frequency can be found by Fourier transforming h_a

$$\begin{aligned} H_a(f) &= \frac{k}{R_p}\sum_{i=0}^{N-1}\exp\left(-j2\pi f\left(\frac{R_p}{c} + i\frac{D\sin\Theta}{c}\right)\right) \\ &= \exp(-j2\pi\frac{R_p}{c})\frac{k}{R_p}\sum_{i=0}^{N-1}\exp\left(-j2\pi f\frac{D\sin\Theta}{c}\right)^i \\ &= \frac{\sin(\pi f\frac{D\sin\Theta}{c}N)}{\sin(\pi f\frac{D\sin\Theta}{c})}\exp(-j\pi f(N-1)\frac{D\sin\Theta}{c})\frac{k}{R_p}\exp(-j2\pi\frac{R_p}{c}). \end{aligned} \tag{3.32}$$

The terms $\exp(-j2\pi\frac{R_p}{c})$ and $\exp(-j\pi f(N-1)\frac{D\sin\Theta}{c})$ are constant phase shifts and play no role for the amplitude of the beam profile. Thus, the amplitude of the beam profile is

$$|H_a(f)| = \left|\frac{k}{R_p}\frac{\sin(N\pi\frac{D}{\lambda}\sin\Theta)}{\sin(\pi\frac{D}{\lambda}\sin\Theta)}\right|. \tag{3.33}$$

The beam profile at 3 MHz is shown in Fig. 3.5 for a 64-element array with $D = 1$ mm.

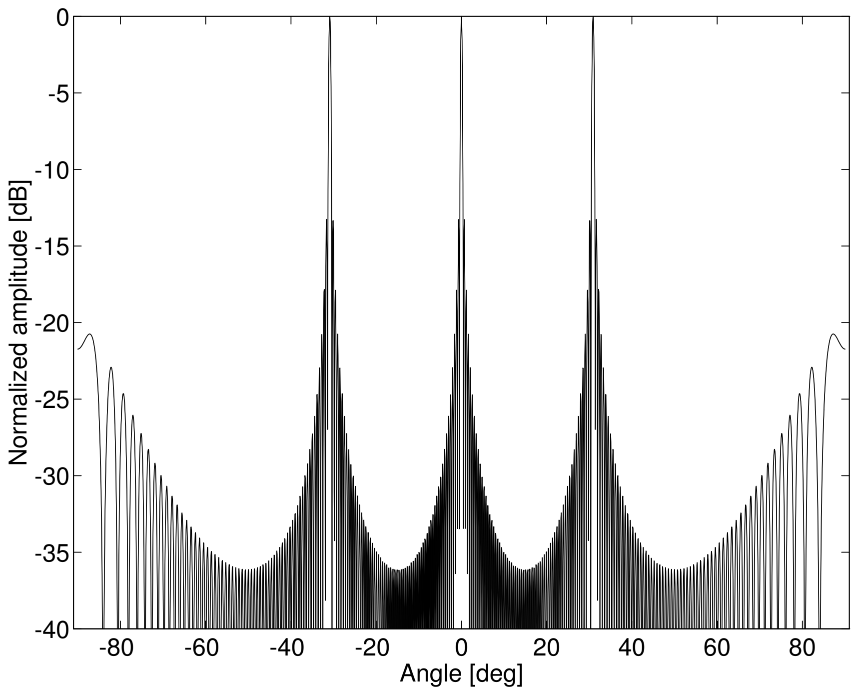


Figure 3.5: Far-field continuous wave beam profile at 3 MHz for linear array consisting of 64 point sources with an inter-element spacing of 1 mm (from [9], Copyright Cambridge University Press).

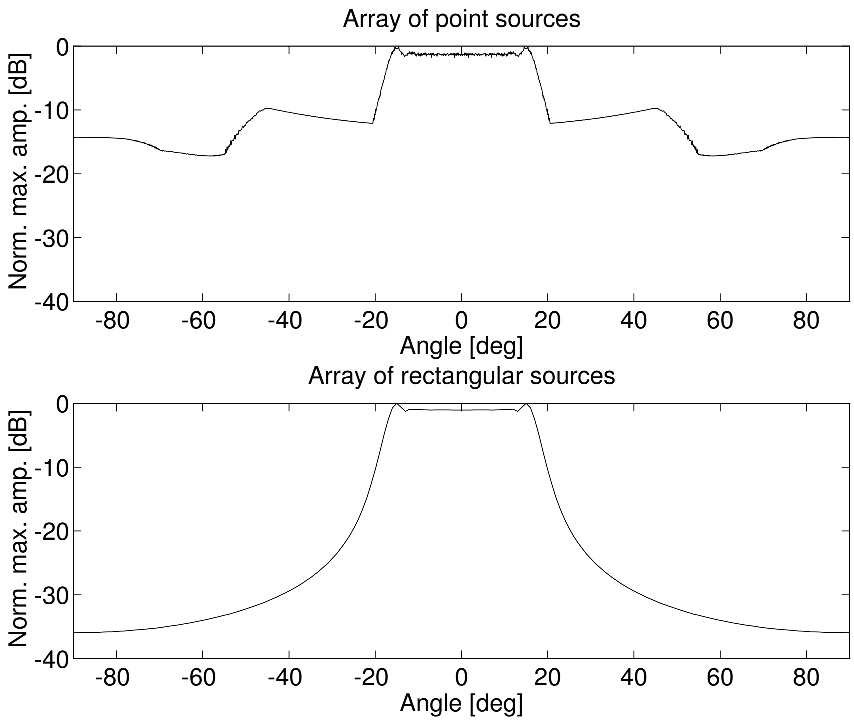


Figure 3.6: Beam profiles for an array consisting of point sources (top) or rectangular elements (bottom). The excitation pulse has a frequency of 3 MHz and the element spacing is 1 mm. The distance to the field point is 100 mm (from [9], Copyright Cambridge University Press).

Several factors change the beam profile for real, pulsed arrays compared with the analysis given here. First, the elements are not points, but rather are rectangular elements with an off-axis spatial impulse response markedly different from a Dirac pulse. Therefore, the spatial impulse responses of the individual elements will overlap and exact cancellation or addition will not take place. Second, the excitation pulse is broad band, which again influences the sidelobes. Examples of simulated responses are shown in Fig. 3.6.

The top graph shows an array of 64 point sources excited with a Gaussian 3 MHz pulse with $B_r = 0.2$. The space between the elements is 1 mm. The maximum of the response at a radial position of 100 mm from the transducer is taken. The bottom graph shows the response when rectangular elements of 1×6 mm are used. This demonstrates the somewhat crude approximation of using the far-field point source CW response to characterize arrays.

Fig. 3.7 shows the different point spread functions encountered when a phased array is used to scan over a 15 cm depth. The array consists of 128 elements each 0.2×5 mm in size, and the kerf between the elements is 0.05 mm. The transmit focus is at 70 mm, and the foci are at 30, 70, and 110 mm during reception. Quite complicated point spread functions are encountered, and they vary substantially with depth in tissue. Notice especially the edge waves, which dominate the response close to the transducer. The edge effect can be reduced by weighting responses from different elements. This is also called apodization. The excitation pulses to elements at the transducer edge are reduced, and this diminishes the edge waves. More examples are shown below in Section 3.6.

3.4 Imaging with arrays

Basically there are three different kinds of images acquired by multi-element array transducers, *i.e.* linear, convex, and phased as shown in Figures 3.8, 3.10, and 3.11. The linear array transducer is shown in Fig. 3.8. It selects the region of

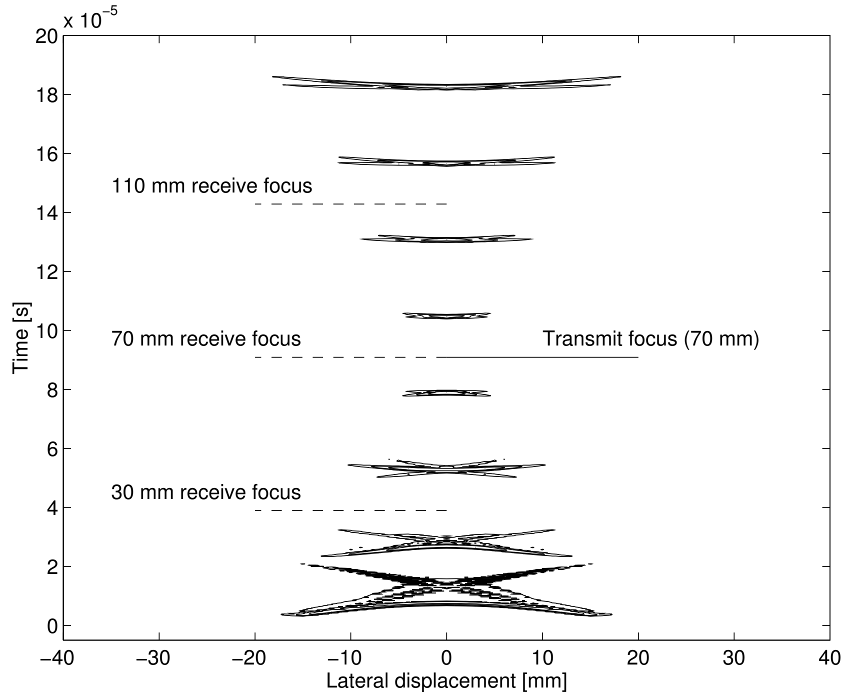


Figure 3.7: Point spread functions for different positions in a B-mode image. Onehundredtwentyeight 0.2 × 5 mm elements are used for generating and receiving the pulsed field. Three different receive foci are used. The contours shown are from 0 to -24 dB in steps of 6 dB relative to the maximum at the particular field point (from [9], Copyright Cambridge University Press).

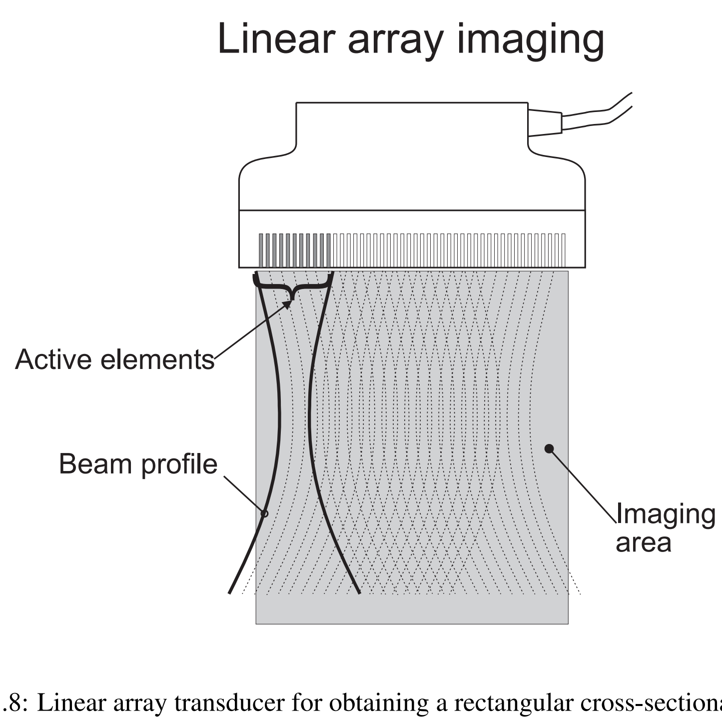


Figure 3.8: Linear array transducer for obtaining a rectangular cross-sectional image.

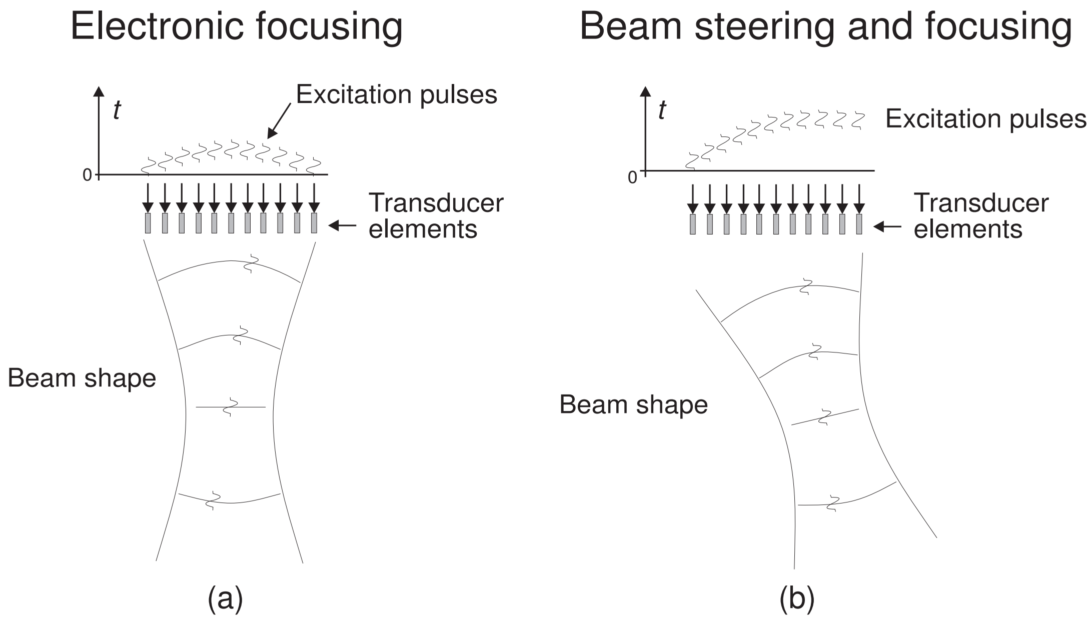


Figure 3.9: Electronic focusing and steering of an ultrasound beam.

investigation by firing a set of elements situated over the region. The beam is moved over the imaging region by firing sets of contiguous elements. Focusing in transmit is achieved by delaying the excitation of the individual elements, so an initially concave beam shape is emitted, as shown in Fig. 3.9.

The beam can also be focused during reception by delaying and adding responses from the different elements. A continuous focus or several focal zones can be maintained as explained in Section 3.2. Only one focal zone is possible in transmit, but a composite image using a set of foci from several transmissions can be made. Often 4 to 8 zones can be individually placed at selected depths in modern scanners. The frame rate is then lowered by the number of transmit foci.

The linear arrays acquire a rectangular image, and the arrays can be quite large to cover a sufficient region of interest (ROI). A larger area can be scanned with a smaller array, if the elements are placed on a convex surface as shown in Fig. 3.10. A sector scan is then obtained. The method of focusing and beam sweeping during transmit and receive is the same as for the linear array, and a substantial number of elements (often 128 or 256) is employed.

The convex and linear arrays are often too large to image the heart when probing between the ribs. A small array size can be used and a large field of view attained by using a phased array as shown in Fig. 3.11. All array elements are used here both during transmit and receive. The direction of the beam is steered by electrically delaying the signals to or from the elements, as shown in Fig. 3.9b. Images can be acquired through a small window and the beam rapidly sweeped over the ROI. The rapid steering of the beam compared to mechanical transducers is of especial importance in flow imaging. This has made the phased array the choice for cardiological investigations through the ribs.

More advanced arrays are even being introduced these years with the increase in number of elements and digital beamforming. Especially elevation focusing (out of the imaging plane) is important. A curved surface as shown in Fig. 3.12 is used for obtaining the elevation focusing essential for an improved image quality. Electronic beamforming can also be used in the elevation direction by dividing the elements in the elevation direction. The elevation focusing in receive can then be dynamically controlled for *e.g.* the array shown in Fig. 3.13.

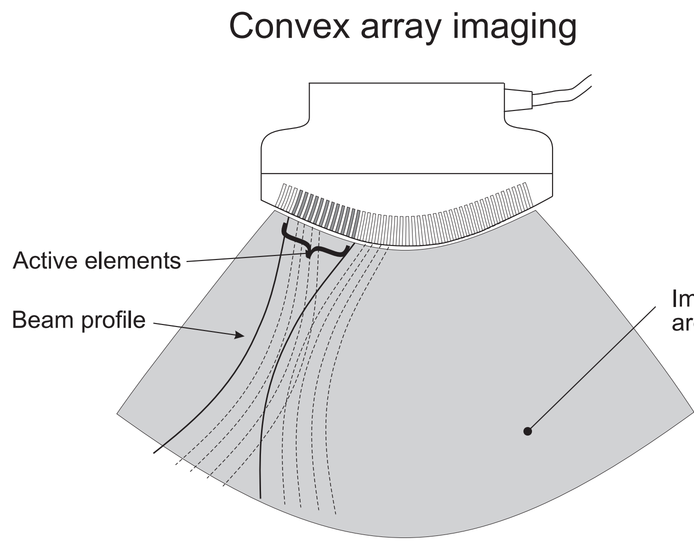


Figure 3.10: Convex array transducer for obtaining a polar cross-sectional image.

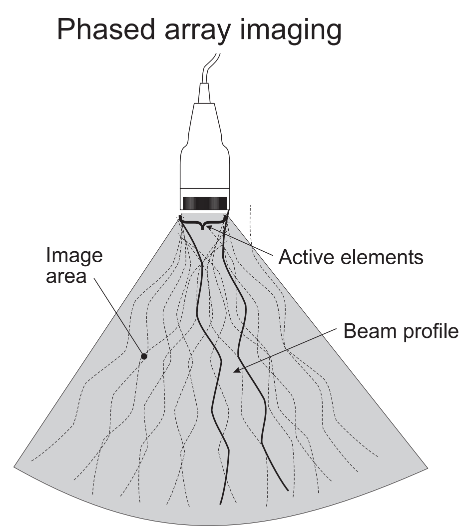


Figure 3.11: Phased array transducer for obtaining a polar cross-sectional image using a transducer with a small foot-print.

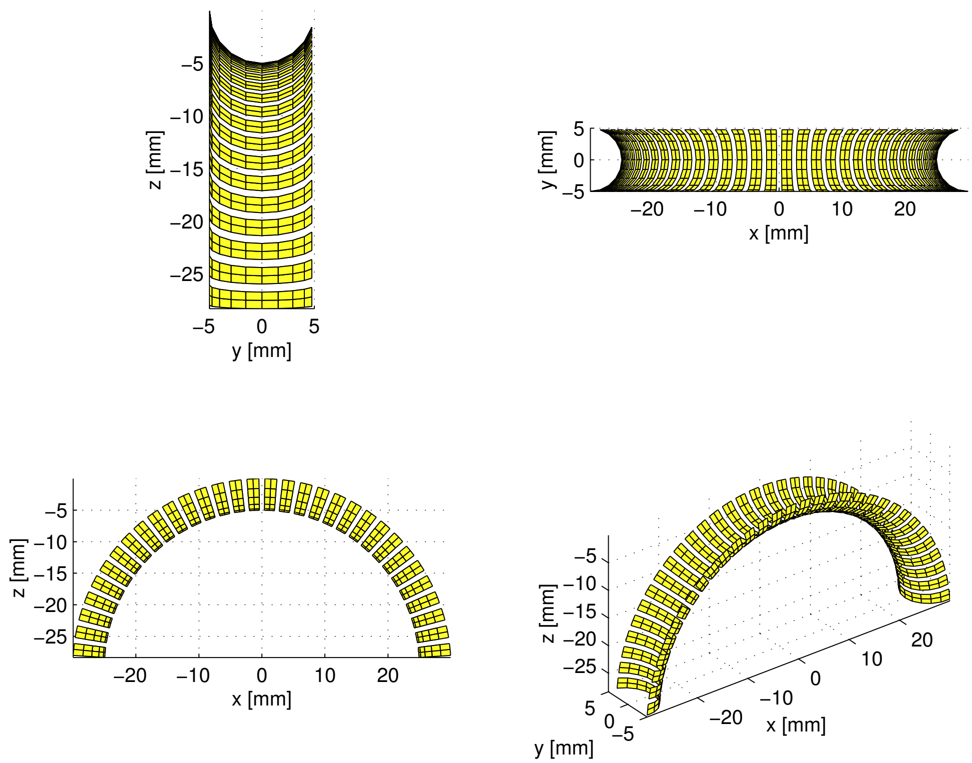


Figure 3.12: Elevation focused convex array transducer for obtaining a rectangular cross-sectional image, which is focused in the out-of-plane direction. The curvature in the elevation direction is exaggerated in the figure for illustration purposes.

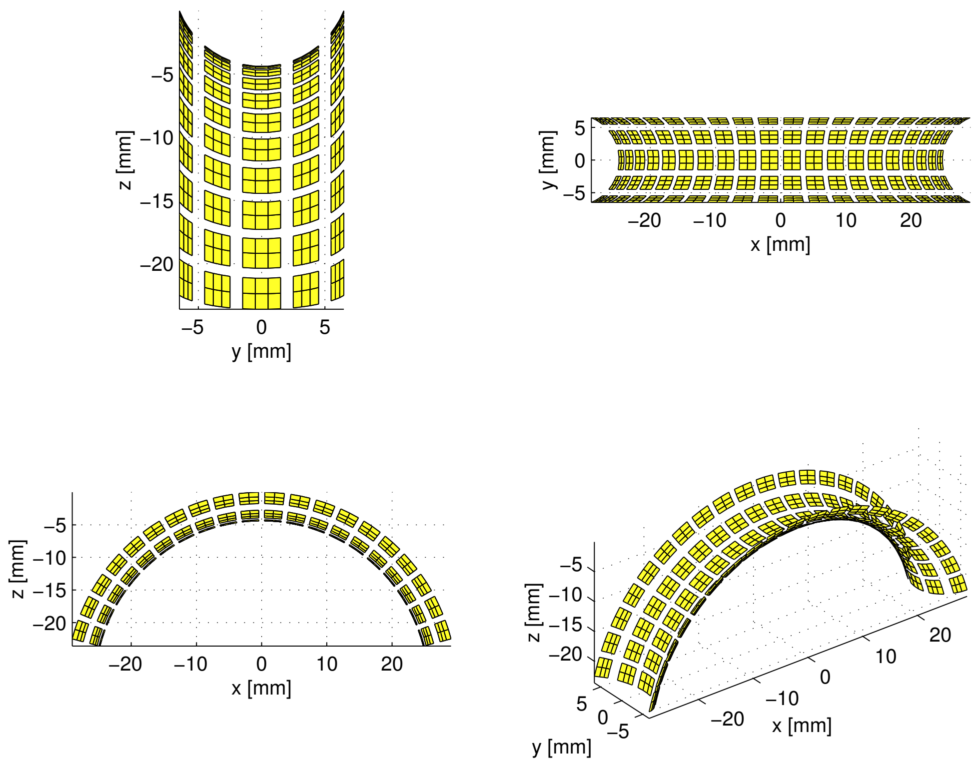


Figure 3.13: Elevation focused convex array transducer with element division in the elevation direction. The curvature in the elevation direction is exaggerated in the figure for illustration purposes

3.5 Simulation of ultrasound imaging

One of the first steps in designing an ultrasound system is the selection of the appropriate number of elements for the array transducers and the number of channels for the beamformer. The focusing strategy in terms of number of focal zones and apodization must also be determined. These choices are often not easy, since it is difficult to determine the effect in the resulting images of increasing the number of channels and selecting more or less advanced focusing schemes. It is therefore beneficial to simulate the whole imaging system in order to quantify the image quality.

The program Field II was rewritten to make it possible to simulate the whole imaging process with time varying focusing and apodization as described in [38] and [21]. This has paved the way for doing realistic simulated imaging with multiple focal zones for transmission and reception and for using dynamic apodization. It is hereby possible to simulate ultrasound imaging for all image types including flow images, and the purpose of this section is to present some standard simulation phantoms that can be used in designing and evaluating ultrasound transducers, beamformers and systems. The phantoms described can be divided into ordinary string/cyst phantoms, artificial human phantoms and flow imaging phantoms. The ordinary computer phantoms include both a string phantoms for evaluating the point spread function as a function of spatial positions as well as a cyst/string phantom. Artificial human phantoms of a fetus in the third month of development and an articial kedney are also shown. The simulation of flow and the associated phantoms will be described in Section 5.5. All the phantoms can be used with any arbitrary transducer configuration like single element, linear, convex, or phased array transducers, with any apodization and focusing scheme.

3.5.1 Simulation model

The first simple treatment of ultrasound is often based on the reflection and transmission of plane waves. It is assumed that the propagating wave impinges on plane boundaries between tissues with different mean acoustic properties. Such boundaries are rarely found in the human body, and seldom show on ultrasound images. This is demonstrated by the image shown in Fig. 3.14. Here the skull of the fetus is not clearly marked. It is quite obvious that there is a clear boundary between the fetus and the surrounding amniotic fluid. The skull boundary is not visible in the image, because the angle between the beam and the boundary has a value such that the sound bounces off in another direction, and, therefore, does not reach the transducer. Despite this, the extent of the head can still be seen. This is due to the scattering of the ultrasound wave. Small changes in density, compressibility, and absorption give rise to a scattered wave radiating in all directions. The backscattered field is received by the transducer and displayed on the screen. One might well argue that scattering is what makes ultrasound images useful for diagnostic purposes, and it is, as will be seen later, the physical phenomena that makes detection of blood velocities possible. Ultrasound scanners are, in fact, optimized to show the backscattered signal, which is considerably weaker than that found from reflecting boundaries. Such reflections will usually be displayed as bright white on the screen, and can potentially saturate the receiving circuits in the scanner. An example can be seen at the neck of the fetus, where a structure is perpendicular to the beam. This strong reflection saturates the input amplifier of this scanner. Typical boundary reflections are encountered from the diaphragm, blood vessel walls, and organ boundaries.

An enlarged view of an image of a liver is seen in Fig. 3.15. The image has a grainy appearance, and not a homogeneous gray or black level as might be expected from homogeneous liver tissue. This type of pattern is called speckle. The displayed signals are the backscatter from the liver tissue, and are due to connective tissue, cells, and fibrious tissue in the liver. These structures are much smaller than one wavelength of the ultrasound, and the speckle pattern displayed does not directly reveal physical structure. It is rather the constructive and destructive interference of scattered signals from all the small structures. So it is not possible to visualize and diagnose microstructure, but the strength of the signal is an indication of pathology. A strong signal from liver tissue, making a bright image, is, *e.g.*, an indication of a fatty or cirrhotic liver.

As the scattered wave emanates from numerous contributors, it is appropriate to characterize it in statistical terms. The amplitude distribution follows a Gaussian distribution [39], and is, thus, fully characterized by its mean and variance. The mean value is zero since the scattered signal is generated by differences in the tissue from the mean acoustic properties.

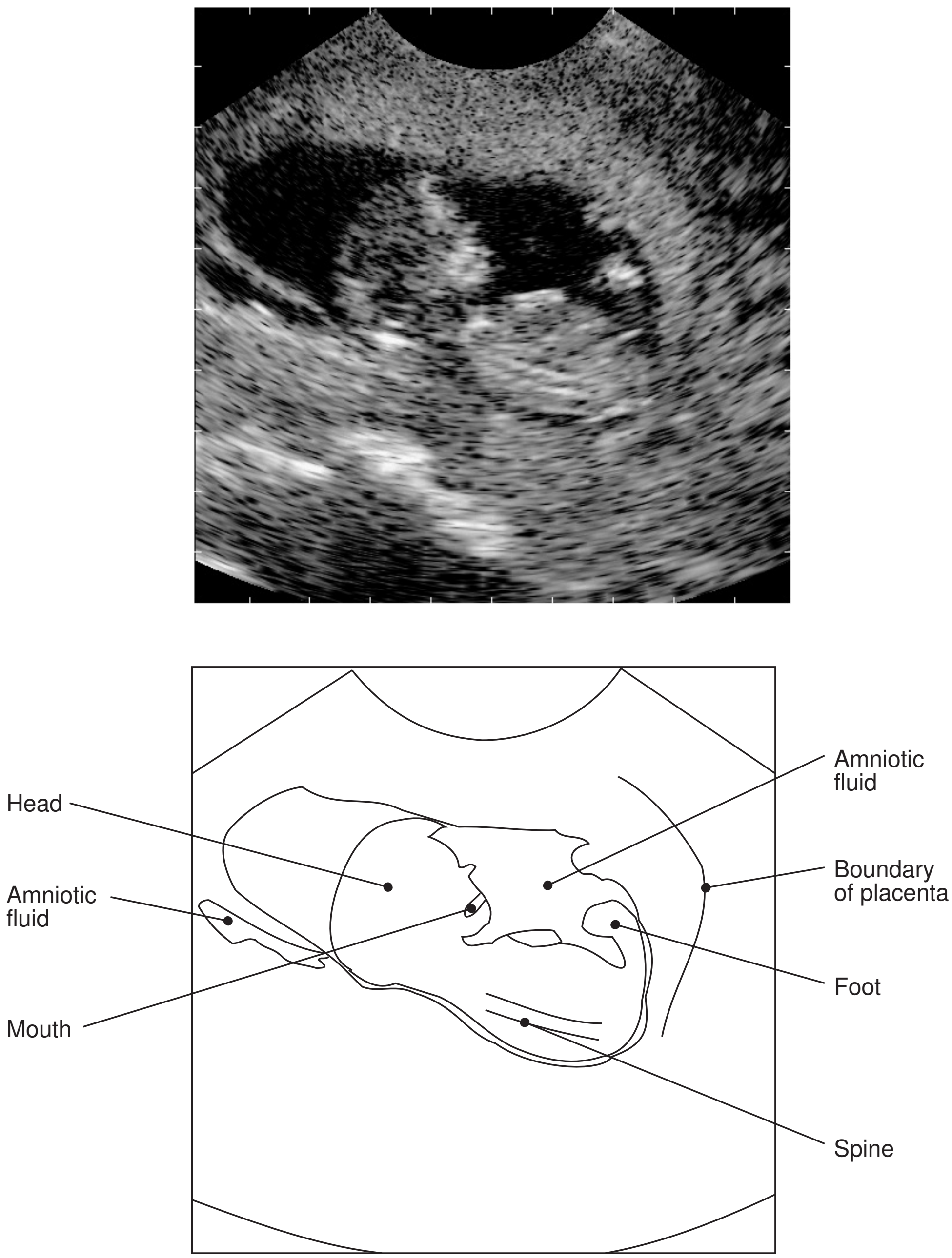


Figure 3.14: Ultrasound image of a 13th week fetus. The markers at the border of the image indicate one centimeter (from [9], Copyright Cambridge University Press).

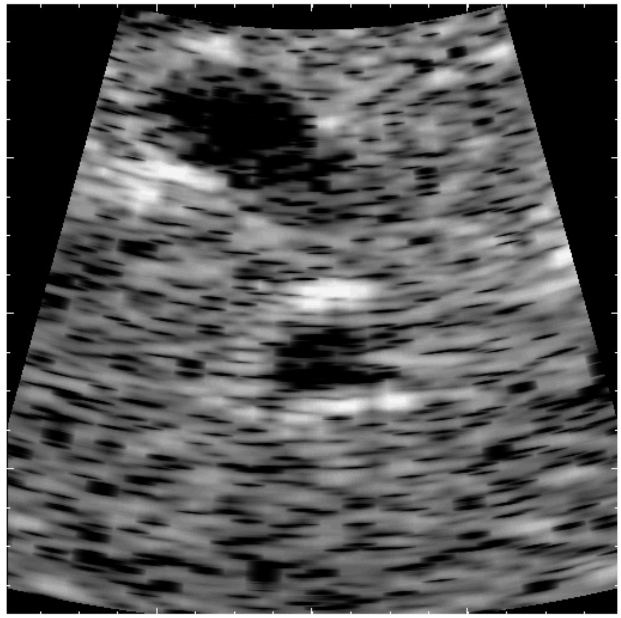


Figure 3.15: 4 × 4 cm image of a human liver from a healthy 28-year-old man. The completely dark areas are blood vessels (from [9], Copyright Cambridge University Press).

Although the backscattered signal is characterized in statistical terms, one should be careful not to interpret the signal as random in the sense that a new set of values is generated for each measurement. The same signal will result, when a transducer is probing the same structure, if the structure is stationary. Even a slight shift in position will yield a backscattered signal correlated with that from the adjacent position. The shift over which the signals are correlated is essentially dependent on the extent of the ultrasound field. This can also be seen from the image in Fig. 3.15, as the laterally elongated white speckles in the image indicate transverse correlation. The extent of these speckle spots is a rough indication of the point spread function of the system.

The correlation between different measurements is what makes it possible to estimate blood velocities with ultrasound. As there is a strong correlation for small movements, it is possible to detect shifts in position by comparing or, more strictly, correlating successive measurements of moving structure, *e.g.*, blood cells.

Since the backscattered signal depends on the constructive and destructive interference of waves from numerous small tissue structures, it is not meaningful to talk about the reflection strength of the individual structures. Rather, it is the deviations within the tissue and the composition of the tissue that determine the strength of the returned signal. The magnitude of the returned signal is, therefore, described in terms of the power of the scattered signal. Since the small structures reradiate waves in all directions and the scattering structures might be ordered in some direction, the returned power will, in general, be dependent on the relative position between the ultrasound emitter and receiver. Such a medium is called anisotropic, examples of which are muscle and kidney tissue. By comparison, liver tissue is a fairly isotropic scattering medium, when its major vessels are excluded, and so is blood.

It is, thus, important that the simulation approach models the scattering mechanisms in the tissue. This is essentially what the model derived in Chapter 2 does. Here the received signal from the transducer is:

$$p_r(\vec{r},t) = v_{pe}(t) \underset{t}{\star} f_m(\vec{r}) \underset{r}{\star} h_{pe}(\vec{r},t) \tag{3.34}$$

where $\underset{r}{\star}$ denotes spatial convolution. v_{pe} is the pulse-echo impulse, which includes the transducer excitation and the electro-mechanical impulse response during emission and reception of the pulse. f_m accounts for the inhomogeneities in the tissue due to density and propagation velocity perturbations which give rise to the scattered signal. h_{pe} is the pulse-echo spatial impulse response that relates the transducer geometry to the spatial extent of the scattered field. Explicitly written out these terms are:

$$v_{pe}(t) = \frac{\rho}{2c^2} E_m(t) \underset{t}{\star} \frac{\partial^3 v(t)}{\partial t^3}, \qquad f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r})}{\rho} - \frac{2\Delta c(\vec{r})}{c}, \qquad h_{pe}(\vec{r}, t) = h_t(\vec{r}, t) * h_r(\vec{r}, t) \tag{3.35}$$

So the received response can be calculated by finding the spatial impulse response for the transmitting and receiving transducer and then convolving with the impulse response of the transducer. A single RF line in an image can be calculated by summing the response from a collection of scatterers in which the scattering strength is determined by the density and speed of sound perturbations in the tissue. Homogeneous tissue can thus be made from a collection of randomly placed scatterers with a scattering strength with a Gaussian distribution, where the variance of the distribution is determined by the backscattering cross-section of the particular tissue. This is the approach taken in these notes.

The computer phantoms typically consist of 100,000 or more discrete scatterers, and simulating 50 to 128 RF lines can take several days depending on the computer used. It is therefore beneficial to split the simulation into concurrently run sessions. This can easily be done by first generating the scatterer's position and amplitude and then storing them in a file. This file can then the be used by a number of workstations to find the RF signal for different imaging directions, which are then stored in separate files; one for each RF line. These files are then used to assemble an image. This is the approach used for the simulations shown here in which 3 Pentium Pro 200 MHz PCs can generate one phantom image over night using Matlab 5 and the Field II program.

3.6 Synthetic phantoms

The first synthetic phantom consists of a number of point targets placed with a distance of 5 mm starting at 15 mm from the transducer surface. A linear sweep image of the points is then made and the resulting image is compressed to show a 40 dB dynamic range. This phantom is suited for showing the spatial variation of the point spread function for a particular transducer, focusing, and apodization scheme.

Twelve examples using this phantom are shown in Fig. 3.16. The top graphs show imaging without apodization and the bottom graphs show images when a Hanning window is used for apodization in both transmit and receive. A 128 elements transducer with a nominal frequency of 3 MHz was used. The element height was 5 mm, the width was a wavelength and the kerf 0.1 mm. The excitation of the transducer consisted of 2 periods of a 3 MHz sinusoid with a Hanning weighting, and the impulse response of both the emit and receive aperture also was a two cycle, Hanning weighted pulse. In the graphs A – C, 64 of the transducer elements were used for imaging, and the scanning was done by translating the 64 active elements over the aperture and focusing in the proper points. In graph D and E 128 elements were used and the imaging was done solely by moving the focal points.

Graph A uses only a single focal point at 60 mm for both emission and reception. B also uses reception focusing at every 20 mm starting from 30 mm. Graph C further adds emission focusing at 10, 20, 40, and 80 mm. D applies the same focal zones as C, but uses 128 elements in the active aperture.

The focusing scheme used for E and F applies a new receive profile for each 2 mm. For analog beamformers this is a small zone size. For digital beamformers it is a large zone size. Digital beamformer can be programmed for each sample and thus a "continuous" beamtracking can be obtained. In imaging systems focusing is used to obtain high detail resolution and high contrast resolution preferably constant for all depths. This is not possible, so compromises must be made. As an example figure F shows the result for multiple transmit zones and receive zones, like E, but now a restriction is put on the active aperture. The size of the aperture is controlled to have a constant F-number (depth of focus in tissue divided by width of aperture), 4 for transmit and 2 for receive, by dynamic apodization. This gives a more homogeneous point spread function throughout the full depth. Especially for the apodized version. Still it can be seen that the composite transmit can be improved in order to avoid the increased width of the point spread function at e.g. 40 and 60 mm.

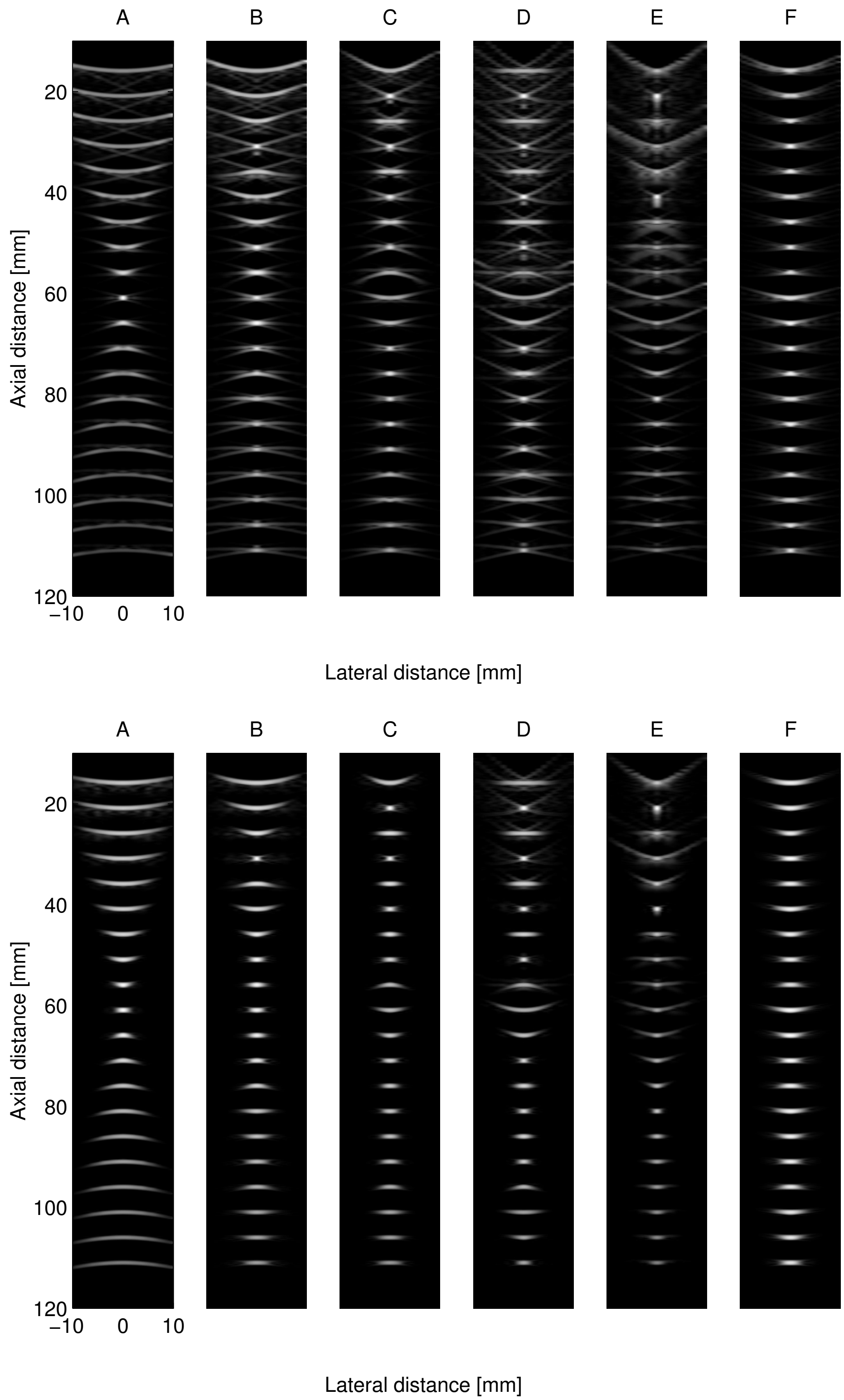


Figure 3.16: Point target phantom imaged for different set-up of transmit and receive focusing and apodization. See text for an explanation of the set-up.

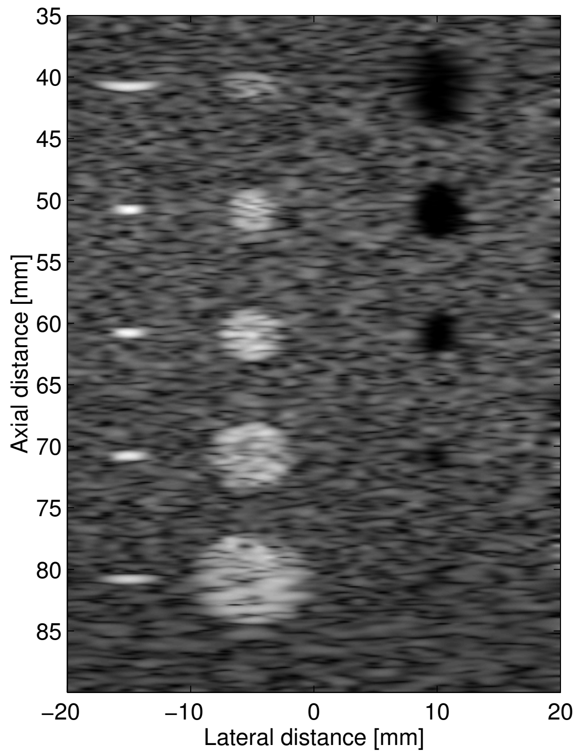


Figure 3.17: Computer phantom with point targets, cyst regions, and strongly reflecting regions.

The next phantom consists of a collection of point targets, five cyst regions, and five highly scattering regions. This can be used for characterizing the contrast-lesion detection capabilities of an imaging system. The scatterers in the phantom are generated by finding their random position within a $60 \times 40 \times 15$ mm cube, and then ascribe a Gaussian distributed amplitude to the scatterers. If the scatterer resides within a cyst region, the amplitude is set to zero. Within the highly scattering region the amplitude is multiplied by 10. The point targets has a fixed amplitude of 100, compared to the standard deviation of the Gaussian distributions of 1. A linear scan of the phantom was done with a 192 element transducer, using 64 active elements with a Hanning apodization in transmit and receive. The element height was 5 mm, the width was a wavelength and the kerf 0.05 mm. The pulses where the same as used for the point phantom mentioned above. A single transmit focus was placed at 60 mm, and receive focusing was done at 20 mm intervals from 30 mm from the transducer surface. The resulting image for 100,000 scatterers is shown in Fig. 3.17. A homogeneous speckle pattern is seen along with all the features of the phantom.

3.7 Anatomic phantoms

The anatomic phantoms are attempts to generate images as they will be seen from real human subjects. This is done by drawing a bitmap image of scattering strength of the region of interest. This map then determines the factor multiplied onto the scattering amplitude generated from the Gaussian distribution, and models the difference in the density and speed of sound perturbations in the tissue. Simulated boundaries were introduced by making lines in the scatterer map along which the strong scatterers were placed. This is marked by completely white lines shown in the scatterer maps. The model is currently two-dimensional, but can readily be expanded to three dimensions. Currently, the elevation direction is merely made by making a 15 mm thickness for the scatterer positions, which are randomly distributed in the interval.

Two different phantoms have been made; a fetus in the third month of development and a left kidney in a longitudinal

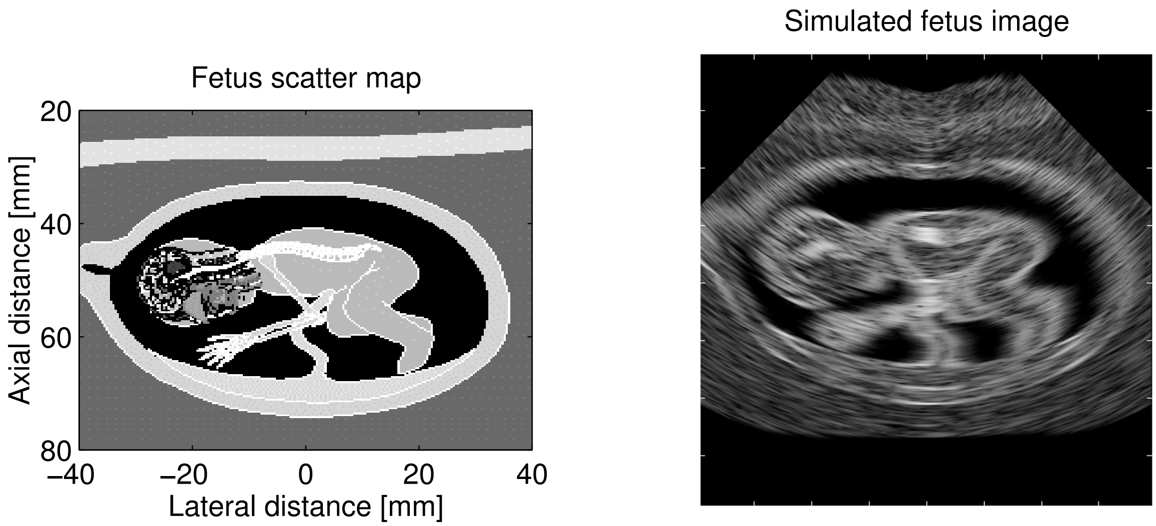


Figure 3.18: Simulation of artificial fetus.

scan. For both was used 200,000 scatterers randomly distributed within the phantom, and with a Gaussian distributed scatterer amplitude with a standard deviation determined by the scatterer map. The phantoms were scanned with a 5 MHz 64 element phased array transducer with $\lambda/2$ spacing and Hanning apodization. A single transmit focus 70 mm from the transducer was used, and focusing during reception is at 40 to 140 mm in 10 mm increments. The images consists of 128 lines with 0.7 degrees between lines.

Fig. 3.19 shows the artificial kidney scatterer map on the left and the resulting image on the right. Note especially the bright regions where the boundary of the kidney is orthogonal to the ultrasound, and thus a large signal is received. Note also the fuzziness of the boundary, where they are parallel with the ultrasound beam, which is also seen on actual ultrasound scans. Fig. 3.18 shows the fetus. Note how the anatomy can be clearly seen at the level of detail of the scatterer map. The same boundary features as for the kidney image is also seen.

The images have many of the features from real scan images, but still lack details. This can be ascribed to the low level of details in the bitmap images, and that only a 2D model is used. But the images do show great potential for making powerful fully synthetic phantoms, that can be used for image quality evaluation.

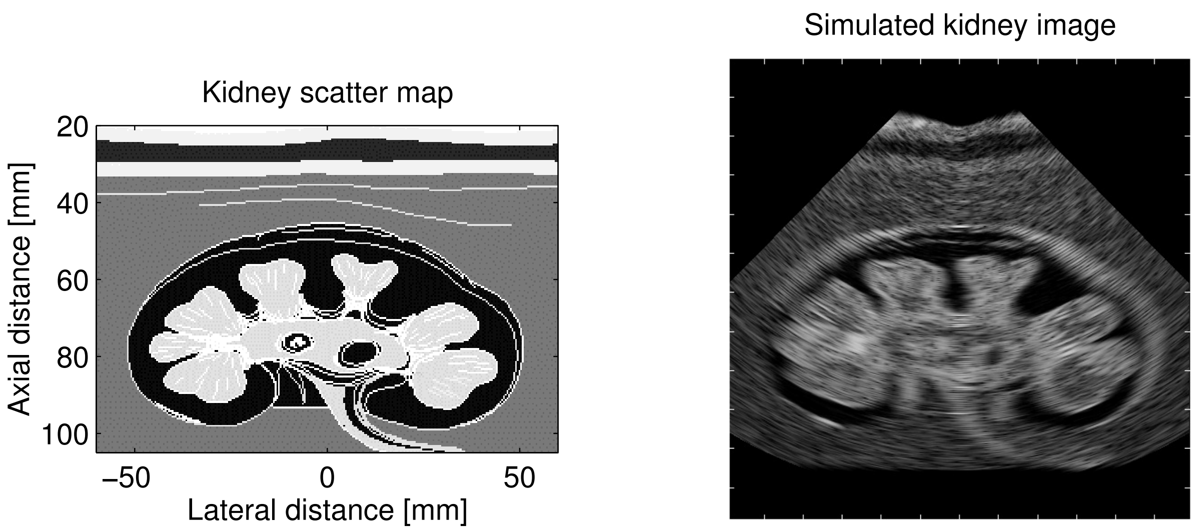


Figure 3.19: Simulation of artificial kidney.

CHAPTER

FOUR

Synthetic aperture ultrasound anatomic and flow imaging

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Synthetic aperture ultrasound imaging

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Synthetic aperture ultrasound imaging

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Abstract

The paper describes the use of synthetic aperture (SA) imaging in medical ultrasound. SA imaging is a radical break with today's commercial systems, where the image is acquired sequentially one image line at a time. This puts a strict limit on the frame rate and the possibility of acquiring a sufficient amount of data for high precision flow estimation. These constrictions can be lifted by employing SA imaging. Here data is acquired simultaneously from all directions over a number of emissions, and the full image can be reconstructed from this data. The paper demonstrates the many benefits of SA imaging. Due to the complete data set, it is possible to have both dynamic transmit and receive focusing to improve contrast and resolution. It is also possible to improve penetration depth by employing codes during ultrasound transmission. Data sets for vector flow imaging can be acquired using short imaging sequences, whereby both the correct velocity magnitude and angle can be estimated. A number of examples of both phantom and in vivo SA images will be presented measured by the experimental ultrasound scanner RASMUS to demonstrate the many benefits of SA imaging.

Keywords: Ultrasound imaging; Synthetic aperture; Vector velocity estimation

1. Introduction

The paper gives a review of synthetic aperture (SA) techniques for medical ultrasound with a description of the current status and the obstacles towards obtaining real-time SA imaging. Synthetic aperture techniques were originally conceived for radar systems in the 1950s and were initially implemented using digital computers in the late 1970s and more advanced techniques were introduced in the late 1980s [1]. There are many similarities between Radar and ultrasound systems, but there are also very significant differences. A SA Radar system usually employs one transmitter and receiver, and the aperture in synthesized by moving the antenna over the region of interest in an airplane or satellite. In medical ultrasound, the array has a fixed number of elements and is usually stationary. The synthesizing is performed by acquiring data from parts of the array to reduce the amount of electronic channels. For Radar, the object is most often in the far-field of the array, whereas the object always is in the near-field of a medical ultrasound system, which complicates the reconstruction. Since the medical array is stationary, it is possible to repeat measurements rapidly, which is not the case for a SA Radar systems. The position between the different elements is also fixed in ultrasound, whereas the deviations from a straight flight path for airplane often have to be compensated for in Radar systems. A vital difference is also that the dynamic range in a Radar image is significantly less than the 40–80 dB dynamic range in ultrasound images.

All these factors affect the implementation of a medical SA ultrasound system and many details have to be changed compared to SA Radar systems to obtain a successful implementation. This paper will describe some of the choices to be made to make a complete SA system that includes vector flow estimation.

Synthetic aperture imaging has been investigated in ultrasonics since the late 1960 and early 1970 [2,3]. In the 1970s and 1980s, it was primarily explored for nondestructive testing (NDT) using a more or less direct implementation of the SA principle, known today as monostatic synthetic aperture imaging [4]. With the introduction of

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transducer arrays in the 1970s, focus was gradually directed towards this application area to pursue real-time implementations [5–7].

Until the beginning of 1990, the idea of applying the synthetic aperture imaging approach for medical ultrasound imaging had only been considered occasionally [3,8]. In 1992, O'Donnell and Thomas published a method intended for intravascular imaging based on synthetic aperture imaging utilizing a circular aperture [9]. To overcome the problem with low SNR and impedance matching between the transducer and receiver circuit, the single element transmission was replaced by simultaneous excitation of a multi-element subaperture. Due to the circular surface of the transducer, the subaperture generated a spherical wave with limited angular extend at each emission, thus, permitting synthetic aperture focusing to be applied. This was the first direct attempt to apply synthetic aperture imaging for medical ultrasound imaging. Since then, the application of multi-element subapertures to increase the SNR of synthetic aperture imaging has been investigated using phased array transducers by Karaman and colleagues for small scale systems [10,11], by Lockwood and colleagues for sparse synthetic aperture systems with primary focus on 3D imaging applications [12,13], and by Nikolov and colleagues for recursive ultrasound imaging [14]. In all cases, the multi-element subaperture was used to emulate the radiation pattern of the single element transmission by applying de-focusing delays in such a way that a spherical wave with limited angular extend was produced. The definition of synthetic transmit aperture (STA) imaging was introduced by Chiao and colleagues in [15]. This paper also considered the feasibility of applying spatial encoding to enable transmission on several elements simultaneously, while separating the individual transmissions in the receiver using addition and subtraction of the received signals. A third approach, which utilizes orthogonal Golay codes to increase the SNR, while transmitting simultaneously on several elements, was also considered by Chiao and Thomas in [16].

The influence of motion in STA imaging and methods for compensation have been investigated in several publications [17–21]. Commonly it is reported that axial motion is the dominant factor causing image quality degradation due to the significantly higher spatial frequency in this dimension. The presented motion estimation methods are generally based on time-domain cross-correlation of reference signals to find the shift in position in the axial dimension. Since tissue motion is inherently three dimensional, it is however likely, that to retain the advantages of STA imaging, at least two dimensional (2D) motion correction to compensate successfully for scan plane tissue motion is required.

2. Conventional ultrasound imaging

Conventional ultrasound images are acquired sequentially one image line at a time. The acquisition rate is, thus, limited by the speed of sound c, and the maximum frame rate f_r for an image with N_l lines to a depth of D is

$$f_r = \frac{c}{2DN_l}. \tag{1}$$

For larger depths and increasing number of lines the frame rate gets progressively lower. The approximate 3-dB resolution of an imaging array consisting of N elements with a pitch of D_p is given by

$$b_{3\,dB} = 0.5\frac{D_i}{ND_p}\lambda = 0.5\frac{D_i}{ND_p}\frac{c}{f_0}, \tag{2}$$

where D_i is focus depth and f_0 is center frequency. Assuming the image to cover the full size of the array and a pitch $D_p = \lambda/2$ then gives a frame rate of

$$N_l = \frac{ND_p}{b_{3\,dB}} = 2\frac{f_0}{D_i c}, \quad f_r = \frac{D_i f_0}{DN^2} \tag{3}$$

for a properly sampled image. Current systems increase the number of active elements in the beamformer and better engineering makes it possible to increase the transducer center frequency for the same penetration depth, which lowers the frame rate, if the image quality has to be maintained.

For flow estimation the problem is increased, since several pulse-echo lines have to be acquired from the same direction in order to estimate the blood velocity [22]. Often 8–16 lines have to be used per estimate and this correspondingly lowers the frame rate. It is 6.4 Hz for a depth of 15 cm, 100 image directions and 8 lines per direction for, e.g., scanning the heart. This is an unacceptable low rate, and the area for estimating the velocity is often limited in conventional systems.

A further problem in conventional imaging is the single transmit focus, so that the imaging is only optimally focused at one depth. This can be overcome by making compound imaging using a number of transmit foci, but the frame rate is then correspondingly decreased.

There are, thus, good reasons for developing alternatives to conventional imaging, where the frame rate and single transmit focusing problems can be solved. One alternative is to use synthetic aperture imaging. It will be shown that this can solve both the frame rate and focusing problem, but it also has several problems associated with it in terms of penetration depth, flow estimation, and implementation. The following sections will address these issues and refer to solutions in the literature.

3. Introduction to synthetic aperture imaging

The basic method for acquiring synthetic aperture ultrasound images is shown in Fig. 1. A single element in the transducer aperture is used for transmitting a spherical wave covering the full image region. The received signals for all or part of the elements in the aperture are sampled for each transmission. This data can be used for making a low resolution image, which is only focused in receive due to the un-focused transmission.

Focusing is performed by finding the geometric distance from the transmitting element to the imaging point and back to the receiving element. Dividing this distance by the speed of sound c gives the time instance $t_p(i,j)$ to take out the proper signal value for summation. For an image point $\vec{r}_p$ the time is, thus:

$$t_p(i,j) = \frac{|\vec{r}_p - \vec{r}_e(i)| + |\vec{r}_p - \vec{r}_r(j)|}{c} \tag{4}$$

where $\vec{r}_e(i)$ denotes the position of the transmitting element i and $\vec{r}_r(j)$ the receiving element j's position. This is done for every point in the resulting image to yield a low resolution image. Combining the low resolution images then results in a high resolution image, since fully dynamic focusing has been performed for all points in the image. The final focused signal $y_f(\vec{r}_p)$ is then:

$$y_f(\vec{r}_p) = \sum_{j=1}^{N} \sum_{i=1}^{M} a(t_p(i,j),i,j) y_r(t_p(i,j),i,j) \tag{5}$$

where $y_r(t,i,j)$ is the received signal for emission i on element j, $a(t_p(i,j),i,j)$ is the weighting function (apodization) applied onto this signal, N is the number of transducer elements, and M is the number of emissions. The transmit focusing is, thus, synthesized by combining the low resolution images, and the focusing calculation makes the transmit focus dynamic for all points in the image. The focus is, therefore, both dynamic in transmit and receive and the highest possible resolution for delay-sum beamforcing is obtained everywhere in the image. Note that the focused signal is a function of space, and that this can be anywhere in the image. Focusing can, thus, be performed in any order and direction, and this will later be used to describe a vector flow system in Section 6. It is also only needed to focus at the points, that are actually shown in the final image as suggested in [23,24]. This, however, necessitates that the complex Hilbert transformed received signal is beamformed to find the instantaneous envelope.

SA imaging makes it possible to decouple frame rate and pulse repetition time, as only a sparse set of emissions can be used for creating a full image. Very fast imaging can, therefore, be made albeit with a lower resolution and higher side-lobes. This can be seen in Fig. 2, where the angular resolution is seen for different number of emissions [25]. A 64 channel fully sampled system was used together with a 5 MHz linear array transducer with a pitch of 0.21 mm. The resolution is determined by the width of the transmitting and receiving aperture and the side-lobe levels are determined by the apodization and the number of emissions.

Very fast imaging at the pulse repetition frequency can be attained by using recursive imaging [14]. The approach uses that the SA acquisition sequence is repeated, so that emission 1 is performed again after all emissions have been made. A full image can be made by combining all emissions, which can be from 1 to M or from 2 to M and 1. The new emission 1 can, thus, replace the old emission 1, which can be done by subtracting the old and adding the new emission. This can be done recursively, which results

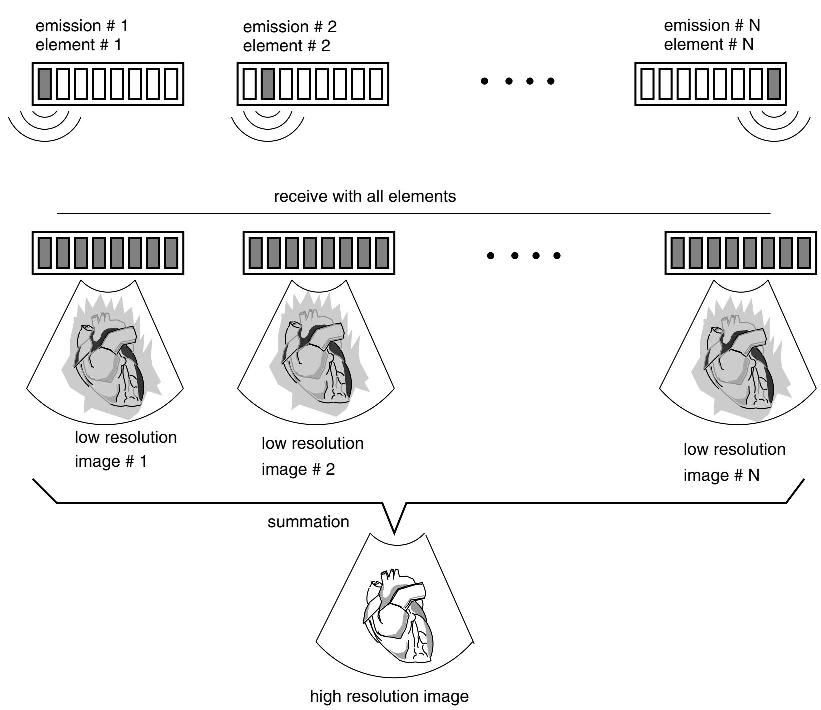

Fig. 1. Basic principle of synthetic aperture ultrasound imaging (from [25]).

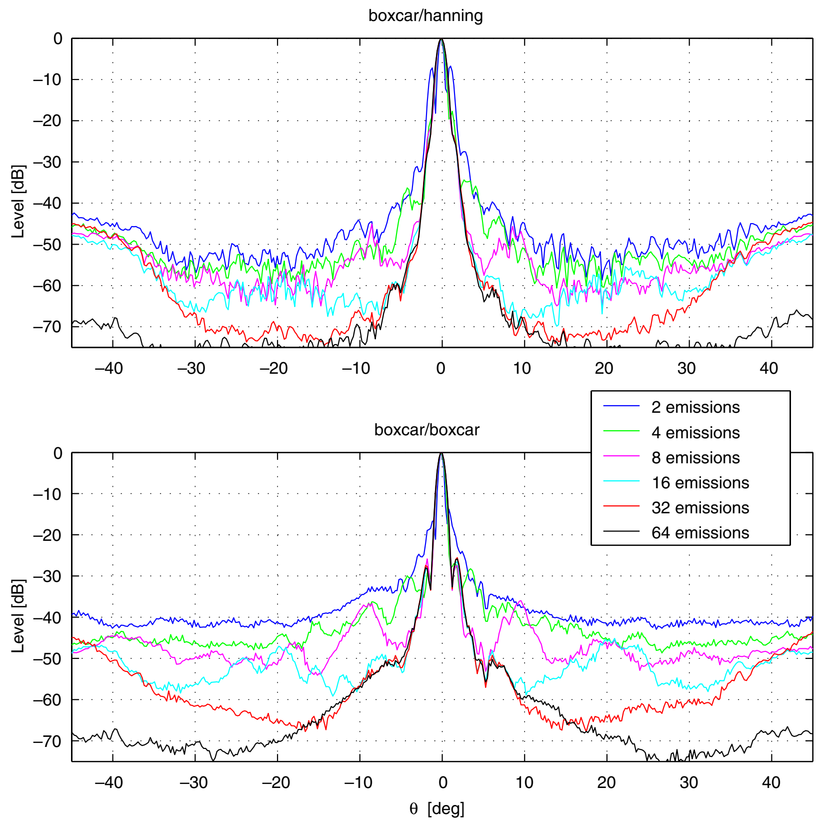

Fig. 2. Angular resolution of a SA imaging system for different number of emissions, when using a boxcar apodization in transmit and a boxcar (bottom) or Hanning apodization in receive (top) (from [25]).

in a new image after every emission. Such an approach can yield very high frame rates and can be used for velocity imaging as described in Section 6.

4. Penetration problem

A major problem in SA imaging is the limited penetration depth, since an un-focused wave is used in transmit and only a single element emits energy. The problem can be solved by combining several elements for transmission and using longer waveforms emitting more energy. Karman et al. [10] suggested combining several elements N_t in transmit, with a delay curve to de-focus the emission to emulate a spherical wave. This can increase the emitted amplitude be a factor of $\sqrt{N_t}$.

It can be combined with using a chirp excitation [26,27] to increase the energy as used in Radar systems [28]. A chirp makes a linear frequency sweep from, e.g., low to high frequencies in the transducer's bandwidth B. Applying a matched filter to the received signal compresses the chirp to a short pulse. The filter is a time reversed version of the pulse and therefore, has the conjugated phase of the chirp. Making the convolution cancels out the phase of the chirp, which makes the resulting signal a linear phase signal and the received signal corresponds to the autocorrelation function of the chirp. Covering the bandwidth of the transducer then gives a resulting pulse or autocorrelation that has a duration proportional to $1/B$. Directly using a rectangular chirp in ultrasound is not possible, as the compressed chirps has temporal side-lobes, which can be as high as -13 dB. This severely limits the contrast of the ultrasound image that has a dynamic range of, e.g., 60 dB. The problem can be solved by applying tapering to the emitted chirp and by applying a window on the matched filter as shown in Fig. 3. The approach was developed in [29–31] that also showed a modest increase in axial resolution of 0.4λ for a gain in signal-to-noise ratio of 10 dB using the modified chirp scheme.

The two approaches can be combined in SA imaging as suggested in [32] to increase the penetration depth. Compared to a conventional ultrasound image the improvement in signal-to-noise ratio is [32]:

$$I_{\mathrm{snr}} = \frac{MN_t}{N_{\mathrm{ct}^2}} \frac{N_{\mathrm{Rs}}}{N_{\mathrm{Rc}}} \frac{T_p}{T_c} \tag{6}$$

where M is the number of emissions for the SA image, N_{Rs} is the number of receive elements and T_p is the duration of the chirp. For the conventional image N_{ct} elements are used in transmit for a pulse of duration T_c seconds and N_{Rc} elements are beamformed in reception. Using the parameters

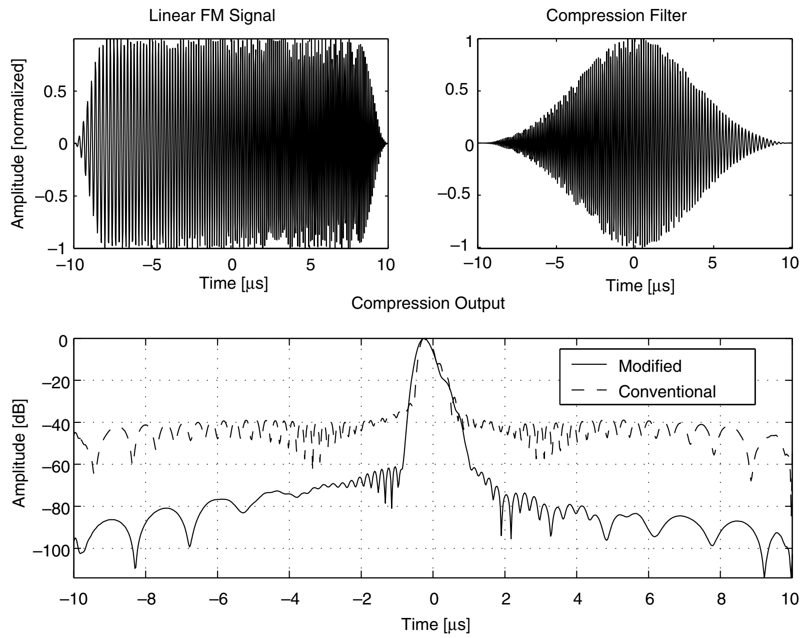

Fig. 3. Top left: Modified linear FM signal with a 7 MHz center frequency and 7 MHz bandwidth. A Tukey window with a duration of 10% has been applied. Top right: Modified compression filter using a Chebychev window with 70 dB relative side-lobe attenuation. Bottom: Compression output for the conventional FM signal (dashed) and the modified FM signal (solid). The effect of a linear array transducer has been introduced in the compression outputs (from [32]).

$M = 96$, $N_t = 33$, $N_{ct} = 64$, $N_{Rs} = 128$, $N_{Rc} = 64$, $T_p = 20\ \mu s$, $T_c = 0.29\ \mu s$ for a 7 MHz system theoretically gives a gain of 17 dB. The actual measurement is shown in Fig. 4 and the calculated gain in Fig. 5. The increase in penetration depth is roughly 4 cm or a 40% gain. An improved focusing scheme has increased the gain by up to 6 dB and further increased the penetration depth [33].

5. Equipment and implementation

The data acquisition in SA imaging is radically different from a normal ultrasound system since data have to be stored for all receiving channels and for a number of emissions. Experiments with SA imaging must, thus, be conducted with dedicated equipment, and only few research groups have access to such systems as no commercial SA research systems are available.

We have developed the remotely accessible software configurable multichannel ultrasound sampling (RASMUS) system specifically tailored for acquiring SA images [34,35]. The system houses 128 transmitter channels that can send arbitrary coded signals with a sampling frequency of 40 MHz and a precision of 12 bits. The coded signals can be different from emission to emission and from channel to channel. It also houses 64 receivers that sample at 40 MHz and 12 bits. They are connected to 1-to-2 multiplexers, so that 128 elements can be sampled over two pulse emissions. The receivers each have associated 256 Mbytes of RAM and can, therefore, sample continuously for more than 3 s to cover a number of heart cycles. The total RAM in the system is more than 24 Gbytes and more than 72 large FPGAs can be used for processing the data [35]. All conventional ultrasound imaging methods can be implemented, but real-time SA imaging is not possible. The data are here stored in the RAM and later processed on a Linux cluster. All the measurements presented in this paper are made with the RASMUS system. A photo of the system and one receiver board for 16 channels is shown in Fig. 6.

6. Flow estimation

In SA imaging, it is possible to focus the received data in any direction and in any order. It does not have to be along the direction of the emitted beam, since the emission is spherical and illuminates the full region of interest. It is, thus, possible to track motion of objects in any direction. This can be used to devise a full vector velocity imaging system.

Conventional ultrasound velocity systems estimate the velocity by finding the shift in position of the scatterers over time [22]. This is done by acquiring lines from the same direction 8 to 16 times and then correlate the data to find the shift in position between lines as either a phase shift [36] or as a time shift [37]. Dividing the spatial shift by the time then gives the velocity. The methods only find the velocity along the ultrasound direction, the standard deviation is often high, and the frame rate is lowered by the number of emissions per direction.

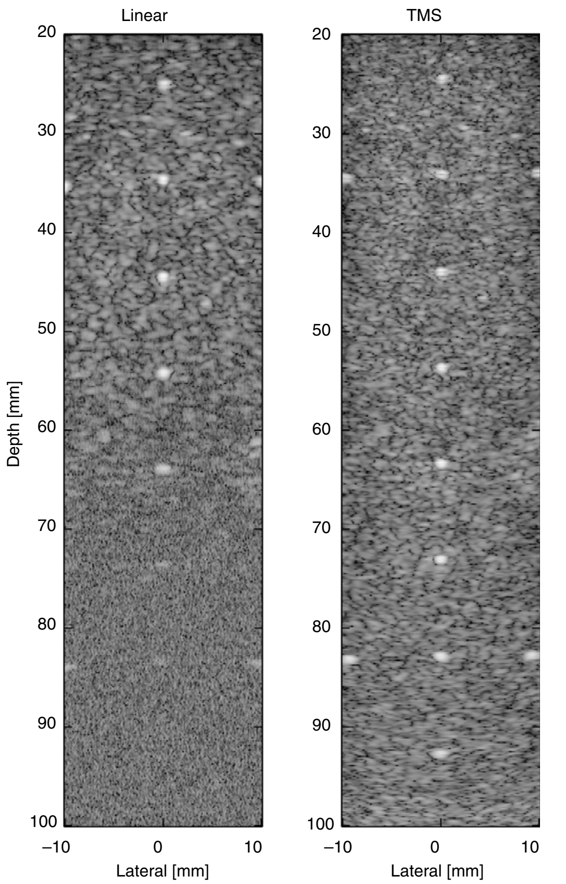

Fig. 4. Measured linear array image (left) and TMS image (right) on a multi-target phantom with 0.5 dB/(cm MHz) attenuation. The scanned section contains twisted nylon wires spaced axially by 1 cm throughout the imaged region. The dynamic range in the images is 50 dB (from [32]).

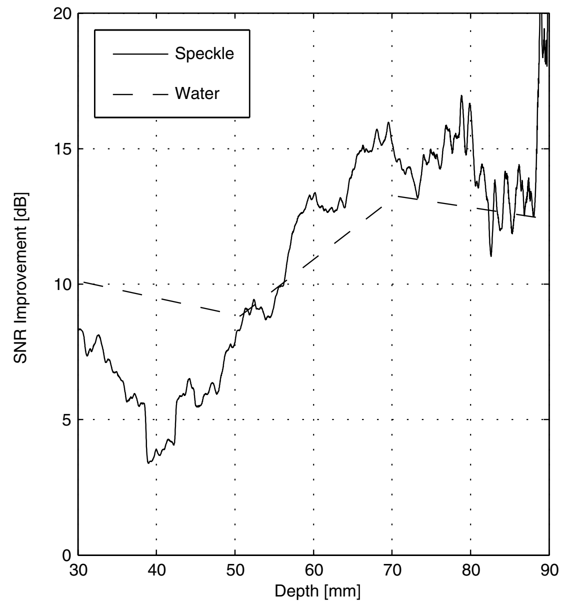

Fig. 5. Calculated SNR improvement obtained by TMS imaging in water (dashed) and in the tissue mimicking phantom (solid) (from [32]).

In SA imaging, the received data can be focused along the direction of the flow as shown in Fig. 7. A short sequence of emissions of $M = 4$–8 is used and the high resolution image lines $y(x')$ are then focused along the flow direction x'. A velocity $\vec{v}$ results in a displacement between high resolution images of

$$\Delta x' = |\vec{v}| M T_{\text{prf}} \tag{7}$$

where T_{prf} is the time between emissions. Data for the first high resolution image line is $y_1(x')$ and the next high resolution image line is $y_2(x') = y_1(x' - \Delta x')$. Cross-correlating the two lines gives a peak at $\Delta x'$ and dividing by MT_{prf} then yields the true velocity magnitude $|\vec{v}|$. This can be done in any direction, also transverse to the normal ultrasound direction of propagation, and the correct velocity magnitude can, therefore, be found [38,39].

The approach has been investigated using a re-circulating flow rig. A 7 MHz linear array with 128 elements was used together with the RASMUS system. A sequence with 8 emissions, using 11 elements and a 20 μs chirp was employed. The flow estimation was performed for 128 emissions and the flow profiles for a fully transverse flow is shown in Fig. 8. The relative standard deviation is 1.2% over the full profile, where a normal system would show a velocity of 0. At 60°, the relative standard deviation is 0.36% [38]. The 128 emissions can also be used for making a full color flow image as shown in Fig. 9 for the flow rig and in vivo for the carotid artery and jugular vein in Fig. 10. The estimates are shown without any averaging or image processing as is used in commercial scanners.

The advantages of the approach is that the velocity can be accurately found in any direction, and that the color flow imaging can be done very fast. Only 128 emissions are needed, where a normal system would need roughly 800 for 100 image directions. The data is also continuously available for all image directions and the velocity can be estimated for as many emissions as the velocity can be assumed constant. The continuous data also makes it easier to perform stationary echo canceling to separate tissue and blood signals, since filters can have any length and initialization can be neglected.

The flow angle must be known before beamforming in the flow direction, and this was in the previous examples estimated from the B-mode image. It can, however, also be estimated from the actual data. For the actual direction the correlation of the data $y_1(x')$ and $y_2(x')$ is highest. For other directions the correlation will drop, since the velocities along that line are different due to the velocity profile of the blood [22]. Calculating the maximum normalized correlation as a function of angle as in [40], thus, gives an index from which the maximum determines the angle as shown in Fig. 11. The function here has a peak at the correct value of 90°. The angle estimates for the profiles

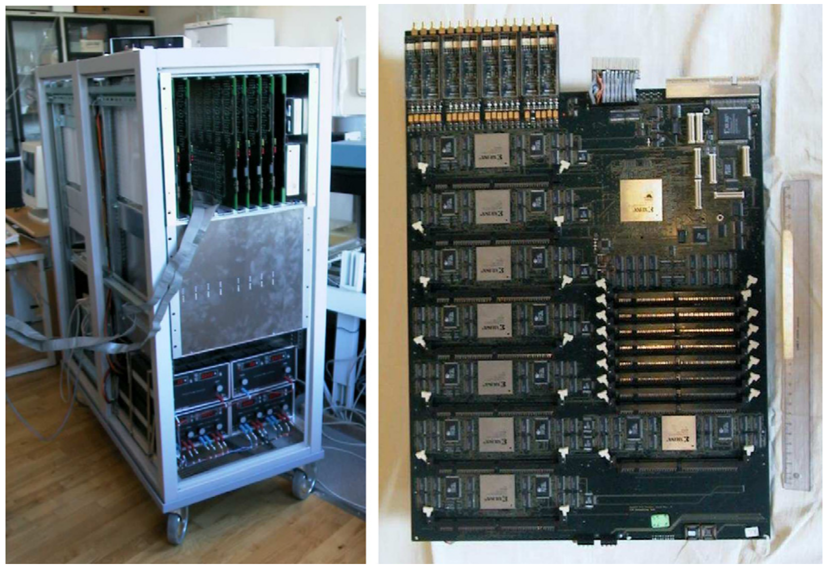

Fig. 6. Photos of the RASMUS scanner (left) with one of the 8 channel receiver boards. The digital part of the system is shown with the 64 receivers in the top cabinet, the 128 transmitters in the middle, and the analog power supplies on the bottom. The analog front-end and transducer plug are at the other side of the 19 in. racks. (from [35]).

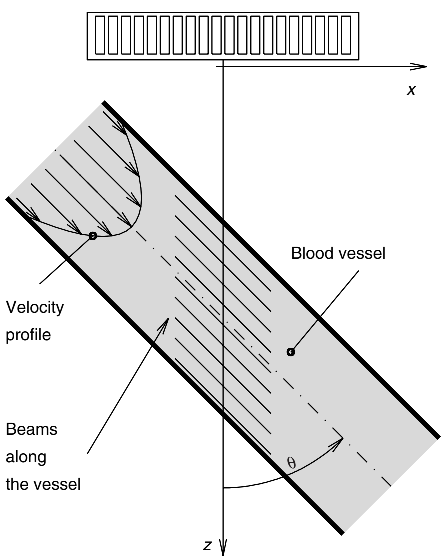

Fig. 7. Beamforming is made along the laminar flow (from [38]).

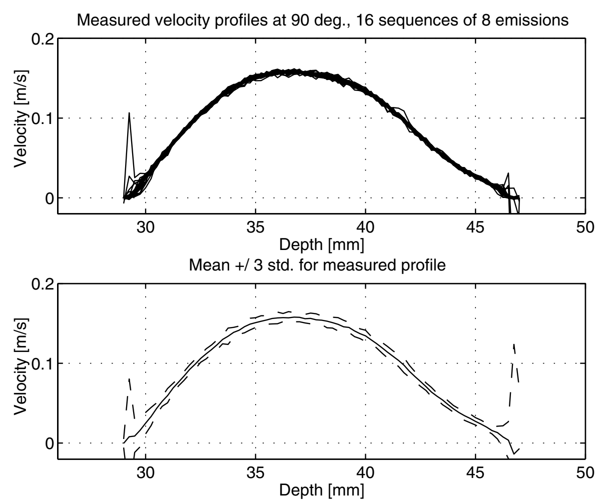

Fig. 8. Estimated profiles from the flow rig at a 90° flow angle. The top graph shows 20 independent profiles estimated and the bottom graph shows the mean profile (solid line) ± 3 standard deviations (dashed lines). From [38].

shown in Fig. 9 are shown in Fig. 12. The mean value is 90.0003° and the standard deviation is 1.32° [40]. The resulting color flow image with arrows indicating direction and magnitude is shown in Fig. 13.

7. Motion compensation

The accurate velocity estimation can also be used for compensating for tissue motion during the SA acquisition process. High quality SA images will often take up to 100 emissions and high tissue velocities will degrade the image quality since the individual low resolution images are not summed in phase. The B-mode sequence can then be inter-spaced with a flow sequence and the tissue velocity can be estimated from this data. Knowing the velocity is then used for correcting the position of the low resolution images that then can be summed in phase. This was suggested in [32,25], where

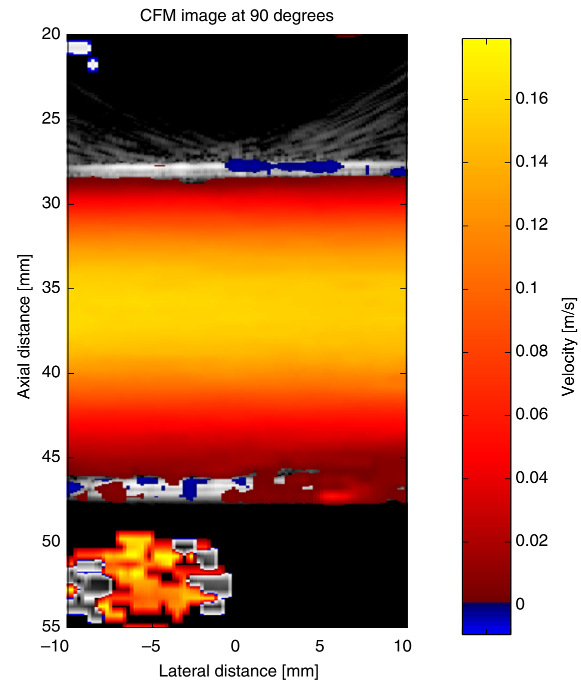

Fig. 9. Synthetic aperture color flow map image of flow rig data at a 90° flow angle obtained using 128 emissions (from [38]). (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

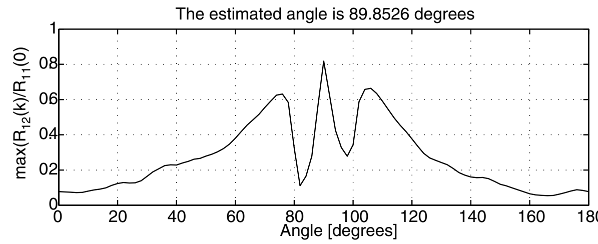

Fig. 11. Normalized maximum correlation as a function of beam formation angle.

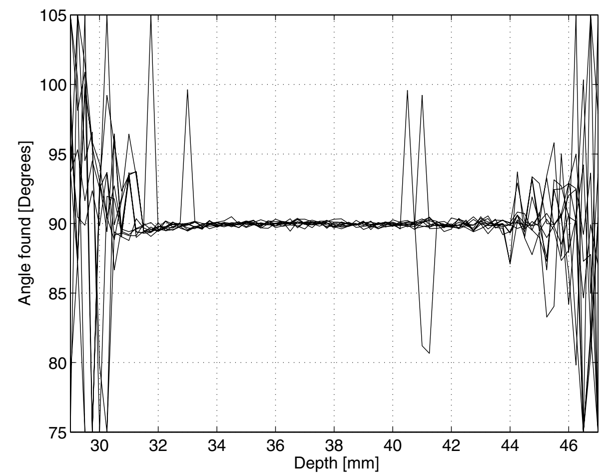

Fig. 12. Estimated velocity angles for a true velocity angle of 90° (from [40]).

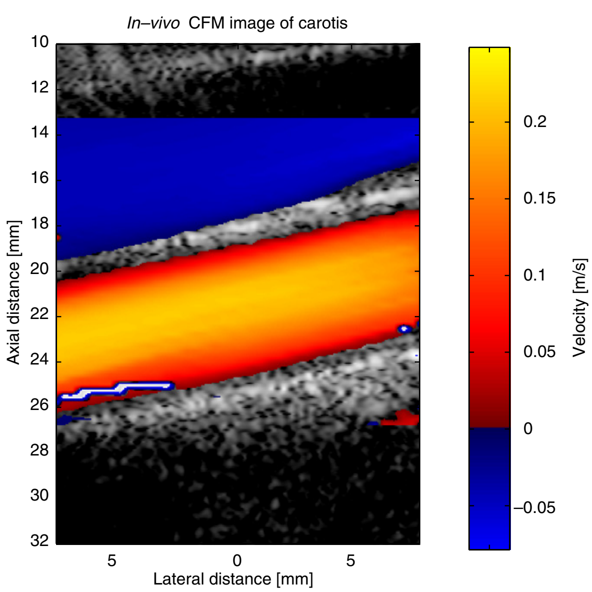

Fig. 10. In vivo color flow map image at a 77° flow angle for the jugular vein and carotid artery. The color scale indicates the velocity along the flow direction, where red hues indicate forward flow and blue reverse flow (from [38]). (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

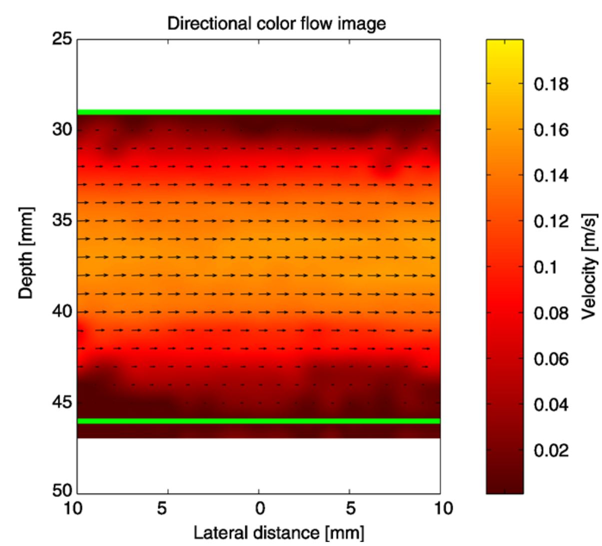

Fig. 13. Vector flow image for data from Fig. 9, when the direction also is estimated. The color show the transverse velocity and the arrows shows velocity magnitude and direction. (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

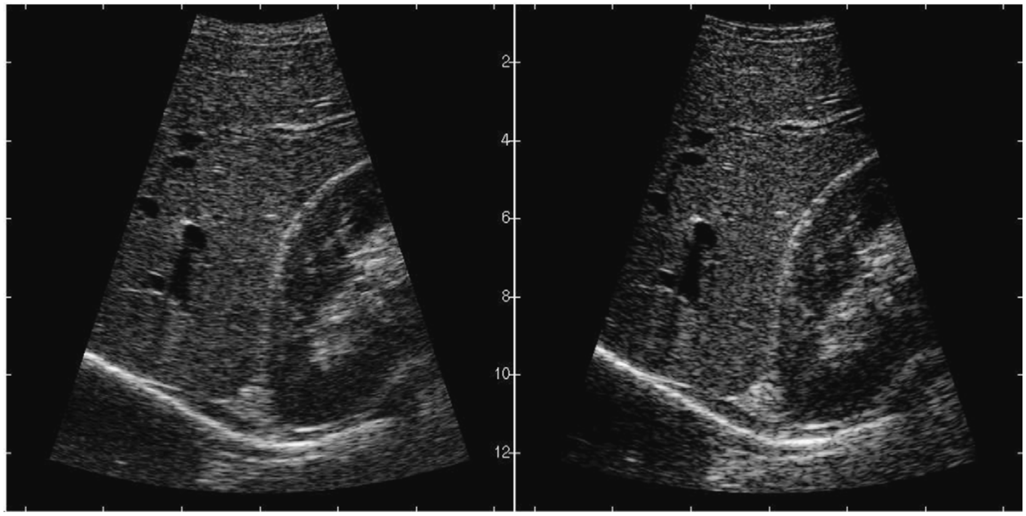

Fig. 14. Examples in vivo images. Left part is the conventional and right is the STA image showing the longitudinal section of right liver lobe showing cross-sections of hepatic vein branches, longitudinal section of a portal a vein branch (upper left part), the kidney, and diaphragm at the bottom.

is was shown that the scheme removes the motion artifacts. Other motion compensation schemes have been studied in [17,19,25].

8. Clinical results

From the previous sections, it can be seen that SA imaging has a large array of advantages compared to conventional ultrasound imaging. It is, however, not clear whether these advantages also translate to the clinical image, and it is, therefore, important to conduct pre-clinical trials to realistically study the performance of SA systems. This can be done with the RASMUS system described in Section 5. It is here possible to acquire in vivo real-time data and then make off-line processing for finding the clinical performance. This has been done in [41,42], where the system was programmed to acquire both a conventional convex array image and a SA image. The sequences were acquired interleaved to have the same region of interest, transducer, and measurement system at the same time. The only change is, thus, the imaging method. An example of such images are shown in Fig. 14 for the liver and right kidney. Seven human volunteers were scanned at two positions for both SA and conventional imaging yielding 28 videos. The sequences were presented to three experienced medical doctors in a double blinded experiment and they were asked to evaluate the images in terms of penetration depth and relative performance between the two images.

The clinical evaluation showed a minute (0.48 cm) but significant ($P = 0.008$) increase in penetration depth using synthetic aperture with coded excitation. Image quality evaluation showed highly significant ($P < 0.001$) improvement in SA images compared to conventional images, which was also expected by the authors due to the apparent improved resolution throughout the SA images.

9. Advanced coded imaging

In the approaches shown in this paper only a single emission center is active at the same time. This limits the emitted energy and the amount of information acquired per emission. It is quite inexpensive to make a transmitter compared to a receiver, and it is, therefore, an advantage to use several emissions simultaneously. Several authors have addressed this problem. Hadamard encoding was suggested in [15] to spatially encode the waveforms, where the Hadamard matrix is multiplied onto the waveforms for the multiple transmissions for a number of transmissions. The Hadamard matrix can also be used for decoding the waveforms, provided the object under investigation is stationary.

This problems was solved in the spread spectrum approach suggested in [43,44]. Here each transmitter is assigned a narrow frequency band. The signals for the individual sources can then be separated using matched filters provided that the bands are disjoint. The high resolution image can then be made by repeating the procedure for all frequency bands for all emitters and then combine all the received signals after filtration. The approach can be used for flow estimation, since the separation is not done over a number of emissions [44].

Methods for even doing single excitation imaging has also be suggested by a number of authors [45–49] and the research fields is still very active. It can potentially lead to a higher penetration and more precise flow images, if the problems with the, e.g., image quality, intensity levels, and computational load can be solved.

10. Summary

This paper has given examples of how medical SA ultrasound imaging can be acquired and processed. It has been shown that problems with penetration depth, flow, and motion can be solved, and that high quality in vivo SA images can be acquired. It has been demonstrated in pre-clinical studies on human volunteers that the SA image resolution and penetration depth are larger than for conventional ultrasound images. Further, the data can be used for vectorial velocity estimation, where both direction and magnitude of the flow vectors can be determined in any direction with a relative standard deviation of a few percent making it possible to construct quantitative SA vector flow systems.

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SA velocity estimation:

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Estimation of High Velocities in Synthetic-Aperture Imaging—Part I: Theory

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Estimation of High Velocities in Synthetic-Aperture Imaging—Part I: Theory

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***Abstract*— This paper describes a new pulse sequence design and estimation approach, which can increase the maximum detectable velocity in synthetic-aperture (SA) velocity imaging. In SA, N spherical or plane waves are emitted, and the sequence is repeated continuously. The N emissions are combined to form a high-resolution image (HRI). Correlation of HRIs is employed to estimate velocity, and the combination of N emissions lowers the effective pulse repetition frequency by N. Interleaving emission sequences can increase the effective pulse repetition frequency to the actual pulse repetition frequency, thereby increasing the maximum detectable velocity by a factor of N. This makes it possible to use longer sequences with better focusing properties. It can also increase the possible interrogation depth for vessels with large velocities. A new cross-correlation vector flow estimator is also presented, which can further increase the maximum detectable velocity by a factor of 3. It is based on transverse oscillation (TO), a preprocessing stage, and cross-correlation of signals beamformed orthogonal to the ultrasound propagation direction. The estimator is self-calibrating without estimating the lateral TO wavelength. This paper develops the theory behind the two methods. The performance is demonstrated in the accompanying paper for convex and phased array probes connected to the synthetic aperture real-time ultrasound system scanner for parabolic flow for both conventional and SA imaging.**

***Index Terms*— Synthetic aperture, ultrasound imaging, velocity estimation.**

I. Introduction

SYNTHETIC aperture (SA) velocity estimation was introduced in 2001 [1], [2]. Here, a repeated sequence of diverging emissions was used for reconstructing a continuous imaging sequence usable for velocity and vector velocity estimation. This resulted in highly accurate velocity estimates using directional beamforming [3] with relative standard deviations (SDs) down to 0.36% and very fast *in vivo* velocity images [2]. Only 24 emissions were used for imaging the carotid artery potentially yielding frame rates up to 10 kHz at a pulse repetition frequency f_{prf} of 24 kHz. Such continuous sequences can also be used with plane wave imaging, and the possibility of infinite observation times has been used to estimate very slow flow in the brain [4], [5] and to derive functional brain activity images. The main possibilities and advantages have been demonstrated by a number of research groups and are described in review papers [6]–[8].

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There is, however, one disadvantage of SA as imaging sequences with a number of emissions have to be acquired to attain a high resolution and low sidelobes. The effective pulse repetition frequency $f_{\text{prf,eff}}$ is the emissions pulse repetition frequency f_{prf} divided by the emissions sequence length N, i.e., $f_{\text{prf,eff}} = f_{\text{prf}}/N$. The highest velocity detectable is directly proportional to $f_{\text{prf,eff}}$ and is, therefore, reduced by a factor N, which can lead to erroneous estimates for large velocities in the major arteries or in the heart. The solution is often to employ a very high f_{prf}, which generates massive amounts of data, precludes the investigation of deep vessels, and often creates probe temperature problems and limitations on the emitted fields.

Several factors influence the resolution limit and contrast for images, which also affects the velocity range possible to estimate. The resolution is determined by the F-number of the system, which is determined by the width of the aperture or rather the number of elements combined in reception, and the spread of the emissions in either angle for plane wave emissions or space for spherical emissions. Having a high number of emissions and receiving elements yields a good contrast and resolution as demonstrated in [9] for plane waves and in [10] for spherical waves. A long sequence will, however, reduce the effective $f_{\text{prf,eff}}$, and the effective frame rate of fully independent images is also reduced by a factor of N. In velocity imaging, the maximum detectable velocity is usually proportional to $\lambda f_{\text{prf}}/4$ for phase estimation methods before aliasing takes place [11], where λ is the wavelength. The maximum detectable velocity is, thus, also reduced by N, and obtaining both a high contrast for separating out adjacent vessels and a high-velocity range seems unbreakable in SA velocity imaging. A choice must, therefore, be made between looking at low-velocity flow in small vessels with a long sequence or having a shorter sequence for estimating fast flow as described in [12]. One approach to break the aliasing limit is to use cross-correlation methods [13]–[15] rather than the autocorrelation method [16] limited to motions within $\pm\lambda/2$. The largest velocities are, however, still limited by rapid decorrelation of the data. An other approach is to use a staggered pulse repetition method for ultrafast imaging of high velocities [17], which is restricted to be used for antialiasing in phase shift estimators.

This paper describes two methods for increasing the aliasing limit to obtain both a high contrast using a long sequence and, at the same time, obtain a high aliasing limit. The first improvement is a new sequence design presented in Section II,

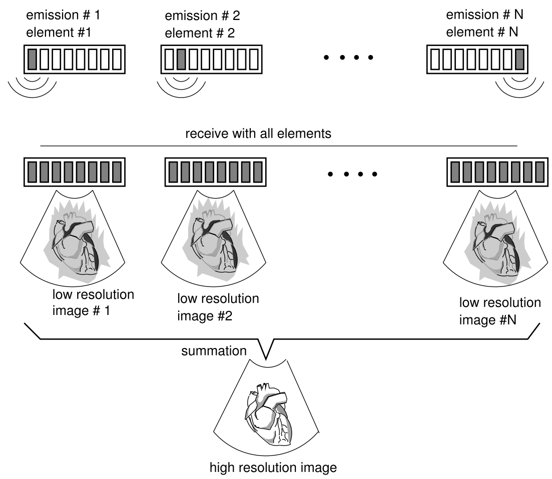

Fig. 1. Visualization of SAI. Spherical waves are emitted from a number of elements and the scattered signals are received on all the elements. An LRI is beamformed for each emission, and they are combined to yield an HRI with dynamic focusing in both transmit and receive (from [1]).

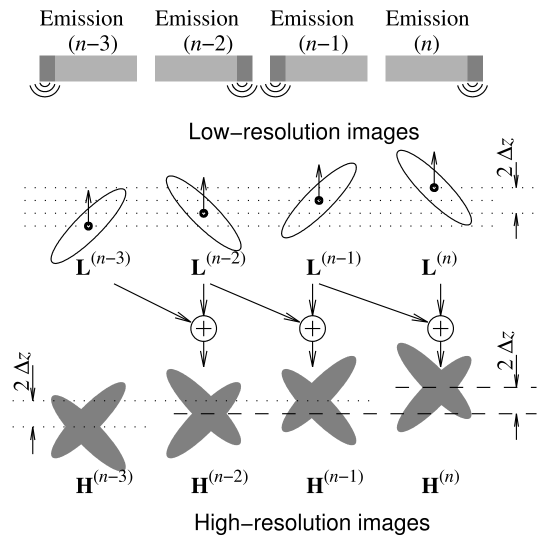

Fig. 2. SA sequence used for flow estimation using a two-emission sequence. Emitting virtual sources (top row). PSF for the LRIs (middle row). Resulting HRIs (bottom row). Similar HRIs can be correlated to estimate the velocity and the repeated sequence yields continuous data for the whole image (from [2]).

which maintains the highest possible $f_{\mathrm{prf,eff}}$ equal to f_{prf}. This maintains a high peak detectable velocity with a reduced amount of data and makes interrogation of deep-lying vessels with a high peak velocity possible. The approach can be combined with any velocity estimator.

Second, a new estimator for the transverse oscillation (TO) approach is introduced [18]–[20], where cross-correlation is used. It can increase the detectable maximum velocity by a factor of 3 breaking the aliasing limit as demonstrated in the accompanying paper [21]. The lateral oscillation period can also be controlled dynamically during receive processing to increase or lower it depending on the flow velocities investigated. The estimator is derived in Section IV.

The accompanying paper [21] presents the Field II simulations [22], [23] and measurements using the synthetic aperture real-time ultrasound system (SARUS) experimental scanner [24] for revealing the performance of the methods.

II. Synthetic-Aperture Flow Imaging

SA imaging (SAI) insonifies a whole region of interest using spherical waves [6], [25] as illustrated in Fig. 1. A virtual source in the form of a spherical or plane wave is emitted. The spherical virtual source can have its origin behind or in front of the array and can use one or a number of elements combined [26]–[28]. A plane emission is defined by its tilt angle compared to the array, and all elements are usually needed in transmit. A low-resolution image (LRI), $L^{(1)}$, is beamformed after each emission, and this is dynamically focused during the receive beamforming based on the placement of the transmitted wave. A new wave is then emitted and the received data beamformed to yield $L^{(2)}$. The process is repeated for all N emissions in the sequence and all LRIs are combined to yield a high-resolution image (HRI), $H^{(N)}$, which is dynamically focused in both transmit and receive. The image is, thus, reconstructed over a number of emissions, and will, therefore, be affected by motion of the interrogated volume.

This is depicted in Fig. 2, which shows the point spread functions (PSFs) for the individual LRIs and the corresponding HRIs for a short two-emission sequence. The scatterer imaged is moving toward the probe by a distance of $\Delta z = v_z T_{\mathrm{prf}} = v_z / f_{\mathrm{prf}}$ between pulse emissions, where v_z is the axial velocity. The PSFs for the LRI are tilted toward the emitting source and are, therefore, different for each LRI in the sequence. The combined HRIs are also different, but it should be noted that the only difference for the same combination of LRIs $L^{(n-3)} + L^{(n-2)} = H^{(n-2)}$ and $L^{(n-1)} + L^{(n)} = H^{(n)}$ is the shift in position. These two HRIs can, therefore, be correlated to find the motion, and this is the key feature used in SA and plane wave flow imaging as was noticed and introduced by Nikolov and Jensen [1], [2] and Nikolov [25].

This ordering of the processing yields continuous data everywhere in the image, which makes it possible to track targets continuously and have very long echo canceling filters and averaging over as long time as the correlation functions are coherent [2], [29], [30]. The velocity can, thus, be found from any of the methods mentioned in [31]. Beamforming can also be performed in any direction and the flow can thus be tracked to minimize decorrelation effects from velocity gradients.

The standard method for SA flow imaging is shown in Fig. 3. A single HRI is created from the same colored block of LRIs and the correlation for finding velocities is between HRIs with a time difference of $T_{\mathrm{prf}} N$. The averaging is across a number of HRI correlations to yield a low SD velocity estimate.

III. Emission Sequence Design

The largest velocity detectable for both spherical and plane wave synthetic aperture focusing (SAFs) $v_{z,\max}$ is determined by the time interval between the two received signals that are

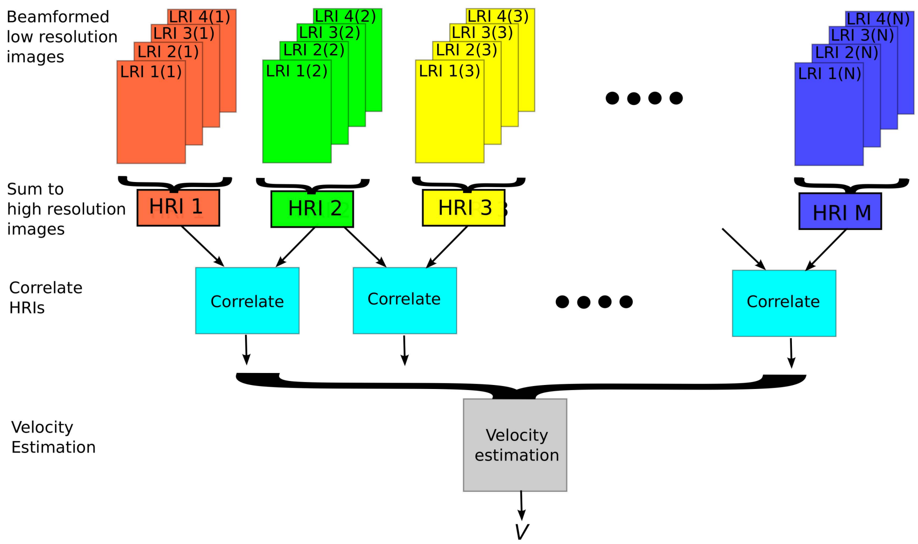

Fig. 3. Traditional SA sequence for velocity estimation. HRIs are created in blocks making the effective $f_{\text{prf,eff}}$ low. LRI 1(2) denotes LRI for emission 1 in high-resolution sequence 2 (first emission in the green block).

correlated and is for an autocorrelation (phase shift) estimator given as

$$v_{z,\max} = \frac{\lambda}{4} f_{\text{prf,eff}}. \tag{1}$$

Keeping $f_{\text{prf,eff}}$ high, thus, ensures the highest detectable velocity. Ideally, the emissions for velocity estimation should be adjacent in time. This is precluded in a SAF system with a sequence length N so $f_{\text{prf,eff}} = f_{\text{prf}}/N$. The sequence should, therefore, be modified, so that HRIs are only one emission apart, at the same time as the sequence length N is maintained. This can be accomplished by interleaving two sequence. The normal sequence is given by

$$\begin{array}{llllll} v_1^{(1)} & v_2^{(1)} & v_3^{(1)} & v_4^{(1)} & \ldots, v_N^{(1)}, \ldots \\ v_1^{(2)} & v_2^{(2)} & v_3^{(2)} & v_4^{(2)} & \ldots, v_N^{(2)} \end{array}$$

where $v_x^{(1)}$ is velocity emission sequence number 1 for spherical or plane wave source x. The source is here a virtual ultrasound source emission [27] or a plane wave in a given direction. Data are then beamformed for all emissions $v_1^{(1)}$ to $v_N^{(1)}$ to create high-resolution data $H^{(1)}$, and for emissions $v_1^{(2)}$ to $v_N^{(2)}$ to create high-resolution data $H^{(2)}$. The two high-resolution data sets are then correlated to estimate the velocity as shown in Fig. 3.

The suggested new sequence interleaves two sequences as shown in Fig. 4

$$v_1^{(1)} \quad v_1^{(2)} \quad v_2^{(1)} \quad v_2^{(2)} \quad \ldots, v_N^{(1)}, \quad v_N^{(2)}.$$

The two high-resolution data sets are beamformed, but $H^{(1)}$ and $H^{(2)}$ are now only one pulse emission apart, and this yields a correlation estimate with the highest possible maximum velocity for both the axial and lateral components. The correlation functions can be averaged over a number of correlation pairs only limited by the acceleration of the flow. The length of averaging is limited by

$$aT < \sigma_v \tag{2}$$

where T is the averaging interval, a is acceleration, and σ_v is the SD on the estimate, which generally is dependent on T. This states that the correlation functions should be averaged as long as the peak position has not moved beyond the precision of the estimate. After this limit, the correlation will start to degrade.

This new sequence gives the optimal data for high-velocity estimation due to the shortest temporal distance between the high-resolution data. It breaks the link between sequence length N and maximum detectable velocity, and it can include long sequences with an optimal resolution and sidelobe level for detecting small vessels. The data sequence is continuous and can, therefore, be averaged over as long time as needed.

It can also be used for recursive SAI [32] as shown in Fig. 5. Here, a new HRI is created after each emission regardless of the imaging length for the fastest possible imaging. The notation *HRI 2(2) 3(2) 4(2) 1(4)* indicates which set of emission is used. The first number is the emission (virtual source) from 1 to N in the sequence. The second number in parentheses is the sequence number used. Therefore, 3(2) is emission number 3 in the second sequence indicated by the light green color. It should be ensured that the same sequence of LRIs is used due to the distortion of the PSF, but after correlation, the functions are similar and can be averaged to increase precision.

The PSF will be affected by the velocity and, thus, the motion between emissions. This will result in a decorrelation of the LRIs, which affects the PSFs as described in [33]. This results in a reduction in SNR and higher sidelobes, which

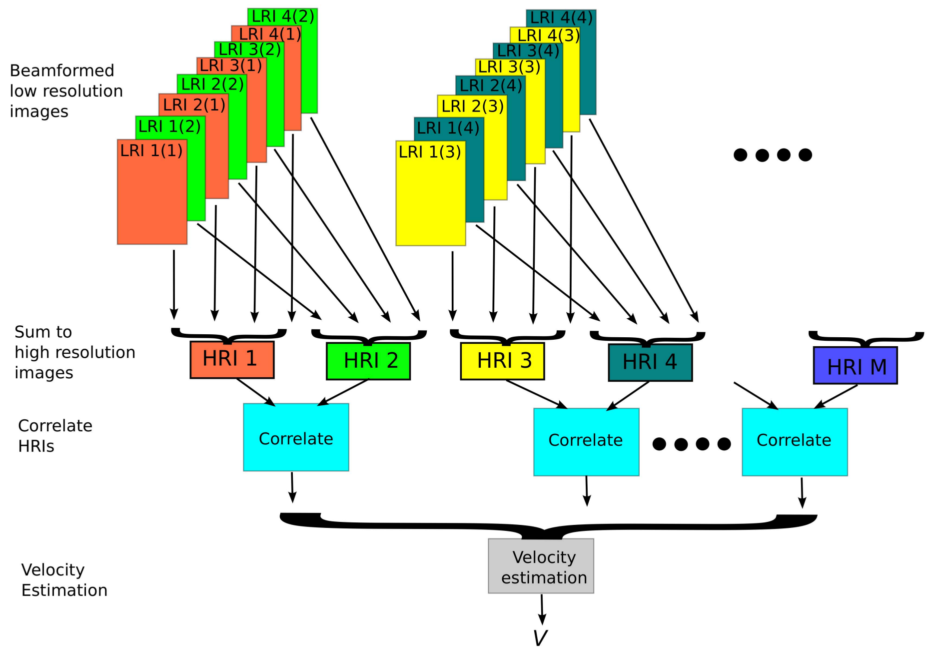

Fig. 4. Interleaved SA sequence where LRIs are repeated to minimize the distance between HRIs. The same colored LRIs are summed to yield one HRI. The effective $f_{\text{prf,eff}}$ is equal to the highest possible value due to the interleaving. Correlations in the blue boxes yield the same correlation function, which are then averaged to improve precision.

affects the velocity estimates. The length of the SA sequence will, therefore, be limited by the time over which the LRIs can be considered correlated. The correlation is affected by the interleaving as the sequence length is essentially doubled, which can lead to a drop in amplitude compared to a noninterleaved sequence. It has, however, been shown that the PSF can be fully recovered, if the velocity vector can be reliably estimated [33], [34], which is more likely for an interleaved sequence. The interleaving can also affect the variance of the estimates, as the length for acquiring data is doubled, thus reducing the averaging in half. The averaging time duration is restricted by the acceleration as given in (2), but often the estimates are more influenced by the echo canceling filter than limited by the averaging duration.

A. Echo Canceling

Echo canceling can be performed on the HRIs using the standard methods for removing stationary tissue signals [11]. It is benefited from having continuous data everywhere in the image [6], and a simple approach is to take the mean value across all HRIs and subtract this from the individual HRIs as used in the accompanying paper. Other more advanced techniques based on, e.g., decompositions [29], [35]–[39] can also be employed, when the continuous data are used as two interleaved high-resolution sequences, if equidistant sampling of the data is a requirement. Finite-impulse response (FIR) and IIR filters can thereby be used, and the continuous data remove limitations from the initial response of the filters. Singular value decomposition (SVD) and other decomposition approaches can easily be adapted to handle the interleaving in one processing stage.

IV. Directional Transverse Oscillation Using Cross-Correlation

The new sequence can be employed with any type of velocity estimation scheme based on correlation functions including autocorrelation [16], [40], cross-correlation [13], speckle tracking [15], directional beamforming [3], and TO [18]. TO introduces a lateral oscillation in the ultrasound field by employing a two-peak apodization waveform during receive beam formation. It can be optimized by focusing a directional signal transverse to the ultrasound propagation direction [directional transverse oscillation (DTO)] [41]. A Hilbert transform along this line is calculated to yield a complex signal usable for finding the sign of the transverse velocity. An autocorrelation estimator has been used to find the transverse velocity. This is limited to shifts less than a quarter lateral wavelength, and the maximum velocity is given by [11]

$$v_{x,\max} = \frac{\lambda_x}{4} f_{\text{prf}} \tag{3}$$

where λ_x is the lateral wavelength. This restriction on maximum velocity coming from employment of phase shift estimation can be alleviated by using a cross-correlation estimator, where the maximum velocity is only limited by the decorrelation of the involved signals [11], [13]. This can be used on the directional signal but at beam-to-flow angles different from 90°, a significant oscillation from the axial motion will introduce a detrimental decorrelation. It can be seen from a model

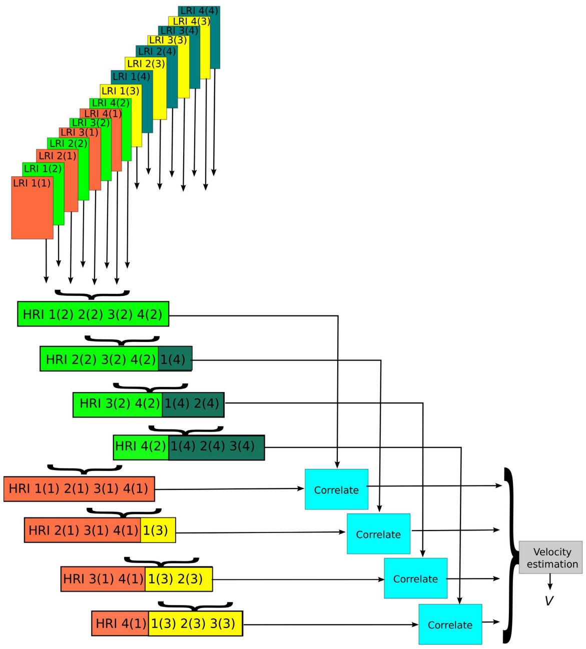

Fig. 5. Processing scheme for recursive SAF, where a new HRI is created after each pulse emission. *HRI 2(2) 3(2) 4(2) 1(4)* indicates which set of emission are used. The first number is the emission (virtual source) and the second number in parentheses is the sequence number used. Therefore, 3(2) is emission number 3 in the second sequence indicated by the light green color.

of the received signal. The received signal is $x(n,k,i)$ and its Hilbert transform along k is $y(n,k,i) = \mathscr{H}_k\{x(n,k,i)\}$, where $\mathscr{H}_k$ denotes the Hilbert transform along k. Here, n is RF sample number, k is sample along the directional signal, and i is emission number. The complex combined signal is [41]

$$\begin{aligned} r_{\text{sq}}(n,k,i) &= x(n,k,i) + j\mathscr{H}_k\{x(n,k,i)\} \\ &= x(n,k,i) + jy(n,k,i). \end{aligned} \tag{4}$$

The received signals are Hilbert transformed in the temporal direction n, and a new directional beamformed signal formed $r_{\text{sqh}}(n,k,i)$ as

$$r_{\text{sqh}}(n,k,i) = \mathscr{H}_n\{x(n,k,i)\} + j\mathscr{H}_n\{y(n,k,i)\}. \tag{5}$$

The signals can be modeled as shown in the equations at the bottom of the page, when assuming monochromatic signals [19], [41]. Here, c is the speed of sound, Δx is the sampling interval along the lateral signal, a is the scattered amplitude, f_s is the temporal sampling frequency, $\Delta z = c/f_s$, and T_{prf} is the time between pulse emissions. The interrogation depth is d, and the two frequencies received from the axial and lateral motions are given by

$$f_p = \frac{2v_z}{c} f_0 = \frac{2v_z}{\lambda}, \quad f_x = \frac{v_x}{\lambda_x}. \tag{6}$$

Two new signals are then formed from

$$\begin{aligned} r_1(n,k,i) &= r_{\text{sq}}(n,k,i) + jr_{\text{sqh}}(n,k,i) \\ r_2(n,k,i) &= r_{\text{sq}}(n,k,i) - jr_{\text{sqh}}(n,k,i). \end{aligned}$$

The combined signals can be written as

$$\begin{aligned} r_1(n,k,i) &= a \cdot \exp\left(j\frac{2\pi}{\lambda}(2v_z i T_{\text{prf}} - n\Delta z - 2d)\right) \\ &\quad \times \exp\left(j\frac{2\pi}{\lambda_x}(v_x i T_{\text{prf}} - k\Delta x)\right) \end{aligned} \tag{7}$$

and

$$\begin{aligned} r_2(n,k,i) &= a \cdot \exp\left(j\frac{2\pi}{\lambda}(2v_z i T_{prf} - n\Delta z - 2d)\right) \\ &\quad \times \exp\left(-j\frac{2\pi}{\lambda_x}(v_x i T_{\text{prf}} - k\Delta x)\right). \end{aligned} \tag{8}$$

Both r_1 and r_2 are influenced by the lateral and axial velocities, and this has previously been separated out using the fourth-order autocorrelation estimators derived in [19] and [41]. For a purely lateral velocity, there is no influence from the axial velocity, and the signals can be cross-correlated to find the spatial shift between two emissions and thereby v_x. For other angles, the estimation process will be distorted to not yield the correct v_x. This is, for example, addressed in directional beamforming [42], [43], which noted the same problem with the transverse correlation approach suggested by Bonnefous [44].

The axial velocity component can be removed by multiplying the two signals as suggested by Anderson [45]. This results in the signal

$$\begin{aligned} r_{\text{mult}}(n,k,i) &= r_1^*(n,k,i) r_2(n,k,i) \\ &= a \cdot \exp\left(-j\frac{2\pi}{\lambda}(2v_z i T_{\text{prf}} - n\Delta z - 2d)\right) \\ &\quad \times \exp\left(-j\frac{2\pi}{\lambda_x}(v_x i T_{\text{prf}} - k\Delta x)\right) \\ &\quad \times \exp\left(j\frac{2\pi}{\lambda}(2v_z i T_{\text{prf}} - n\Delta z - 2d)\right) \\ &\quad \times \exp\left(-j\frac{2\pi}{\lambda_x}(v_x i T_{\text{prf}} - k\Delta x)\right), \\ &= a \times \exp\left(-j\frac{4\pi}{\lambda_x}(v_x i T_{\text{prf}} - k\Delta x)\right) \end{aligned} \tag{9}$$

$$\begin{aligned} r_{\text{sq}}(n,k,i) &= a\frac{1}{2}\left(\exp\left(j2\pi\left(\left(\frac{v_x}{\lambda_x} + \frac{2v_z}{\lambda}\right) i T_{\text{prf}} - \frac{k\Delta x}{\lambda_x} - f_0\frac{n}{f_s} + \frac{2d}{c} f_0\right)\right)\right. \\ &\quad \left. + \exp\left(j2\pi\left(\left(\frac{v_x}{\lambda_x} - \frac{2v_z}{\lambda}\right) i T_{\text{prf}} - \frac{k\Delta x}{\lambda_x} - f_0\frac{n}{f_s} + \frac{2d}{c} f_0\right)\right)\right) \\ r_{\text{sqh}}(n,k,i) &= a\frac{1}{2j}\left(\exp\left((j2\pi\left(\left(\frac{v_x}{\lambda_x} + \frac{2v_z}{\lambda}\right) i T_{\text{prf}} - \frac{k\Delta x}{\lambda_x} - f_0\frac{n}{f_s} + \frac{2d}{c} f_0\right)\right)\right. \\ &\quad \left. - \exp\left(j2\pi\left(\left(\frac{v_x}{\lambda_x} - \frac{2v_z}{\lambda}\right) i T_{\text{prf}} - \frac{k\Delta x}{\lambda_x} - f_0\frac{n}{f_s} + \frac{2d}{c} f_0\right)\right)\right) \end{aligned}$$

where $r_1^*()$ denotes the complex conjugate. The multiplication, thus, removes the influence from the axial velocity, and the resulting signals can be directly correlated to yield the velocity as

$$\begin{aligned} R_{12}(n,m) &= \sum_{i=1}^{N_e}\sum_{k=1}^{N_s} r^*_{\text{mult}}(n,k,i)\, r_{\text{mult}}(n,k+m,i+1) \\ &= \sum_{i=1}^{N_e}\sum_{k=1}^{N_s} a \exp\left(j\frac{4\pi}{\lambda_x}\left(v_x i T_{\text{prf}} - k\Delta x\right)\right) \\ &\quad a \exp\left(-j\frac{4\pi}{\lambda_x}(v_x(i+1)T_{\text{prf}} - (k+m)\Delta x)\right) \\ &= a^2 \exp\left(j2\frac{4\pi}{\lambda_x}(v_x T_{\text{prf}} - m\Delta x)\right) \end{aligned} \tag{10}$$

where N_e is the number of emissions, and N_s is the number of samples in the directional line. This correlation function has a global maximum for $m_p = v_x T_{\text{prf}}/\Delta x$, when a pulsed signal is used. The maximum can be found from the absolute value of the complex correlation function or from the real part of $R_{12}(n,m)$, where the last method gives the most precise determination. The peak value is found as an integer, which limits the precision. It can be increased by using parabolic interpolation by fitting a second-order polynomial to the three points around the peak value. An interpolated peak value is found in [46]

$$m_{\text{int}} = m_p - \frac{\hat{R}_{12}(m_p+1) - \hat{R}_{12}(m_p-1)}{2(\hat{R}_{12}(m_p+1) - 2\hat{R}_{12}(m_p) + \hat{R}_{12}(m_p-1))} \tag{11}$$

to yield the interpolated estimate

$$\hat{v}_{\text{int}} = \frac{m_{\text{int}}\Delta x}{T_{\text{prf}}}. \tag{12}$$

Similar expressions can also be found for the axial cross-correlation estimator.

This new estimator has a number of advantages over the autocorrelation approach. Only knowledge of T_{prf} and Δx is needed to get quantitative results. No calibration estimate of $f_x = 1/\lambda_x$, thus, has to be found as in [41]. Furthermore, the velocity range is not restricted to spatial shifts between $-\lambda_x/2$ and $+\lambda_x/2$ as cross-correlation estimators have no inherent aliasing limit, apart from the decorrelation of signals from transverse beam modulation. This can potentially yield a much higher detectable velocity range, with a reduced beamforming load compared to directional beamforming [42], [43]. This estimator design can also be used for making the transverse spectrum as described in [47].

The cross-correlation limit is determined by the time over which two received signals are correlated, which is determined by the width of the PSF compared to the motion $v_x T_{\text{prf}}$. In SAF, the entire focusing is performed during the receive processing, and this can be adapted to the velocity investigated. The lateral oscillation wavelength can be increased by changing the apodization function to have peaks closer to each other on the virtual aperture for high velocities, or they can be separated more to increase the lateral oscillation frequency for low-velocity estimation. It is also possible to increase the lateral oscillation frequency by using a two-peak apodization in both transmit and receive focusing. An alternative approach is to use full dynamic focusing and then create the lateral oscillation in the frequency domain as suggested in [48] and [49].

V. Conclusion

This paper has described two new methods for increasing the maximum detectable velocity of SA velocity imaging using spherical or plane waves. A new sequence design can increase the velocity limit by a factor N equal to the sequence length, and the new TO directional cross-correlation estimator can break the aliasing limit. It also has a higher aliasing limit by a factor 2–4 than for axial autocorrelation estimation, as the lateral wavelength easily can be made two to four times larger than the axial wavelength [18]. The employment of a cross-correlation estimator instead of the autocorrelation approach further adds a factor of 3 as experimentally shown in the accompanying paper [21]. The combination of all these features makes it possible to estimate lateral velocity components 6 N to 12 N times higher than for axial velocity components in the previous SA velocity sequences. The approach described in [3] used $N = 8$ and $f_{\text{prf}} = 3$ kHz to estimate peak velocities around 0.15 m/s using directional cross-correlation. The aliasing limit for an autocorrelation system would be 0.04 m/s. The same setup could translate to a peak detectable velocity of 4.6–9.2 m/s using the new scheme and estimator at 3 MHz. Increasing f_{prf} to 5 kHz for full cardiac imaging could lead to velocity range above 10 m/s; enough to detect jets from regurgitation in heart valves.

The interleaved approach is not restricted to use with the new DTO cross-correlation estimator but can be used with any of the current velocity estimators used for axial and vectorial velocity estimations [8], [31] and still attain a factor of N increase in maximum detectable velocity.

The accompanying paper [21] investigates the methods using Field II simulations [22], [23] and measurements from the SARUS experimental scanner [24]. It is demonstrated that velocities of 0.5 m/s can be estimated for an f_{prf} of 450 Hz, which translates to 5.6 m/s at $f_{\text{prf}} = 5$ kHz, and in certain cases at $f_{\text{prf}} = 225$ Hz, a velocity of 0.5 m/s could be estimated corresponding to 11.2 m/s at 5 kHz.

The method still has the advantage of continuous data, and the lowest velocity detectable is, therefore, only limited by the duration over which the data can be acquired for the position in the image. The velocity range, therefore, both cover high velocities in the major arteries, at the same time, as low-velocity flow in small vessels can be estimated from the same data.

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CHAPTER

FIVE

Ultrasound Velocity Imaging

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Jørgen Arendt Jensen :

Ultrasound Velocity Imaging

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5.1 Introduction: Blood Velocity Estimation Systems

This Chapter gives an introduction to velocity estimation in medical ultrasound, which is primarily used in cardiovascular imaging, but it can also be applied in *e.g.* strain imaging and tissue motion estimation. The Chapter gives a brief introduction to the human circulation to establish the requirements on blood velocity estimation systems. A simple model for the interaction between ultrasound and point scatterers is derived in Section 5.4 based on the information about scattering from blood in Section 5.3. The model is used in the derivation of velocity estimators based on the signal's frequency content (Section 5.6), phase shift (Section 5.7), and time shift (Section 5.8). These Sections should give a brief introduction to the function of the most prominent methods used in current commercial ultrasound scanners. Newer and more experimental techniques are described in the Sections on vector velocity imaging, synthetic aperture imaging and other applications in Sections 5.9 to 5.12.

The Chapter is necessarily brief in covering the many topics of this field, and the reader is referred to the more in-depth treatment of the topics in [9, 40, 41, 42, 43] along with the references given in the text.

5.2 The human circulation

The major arteries and veins in the human circulation are shown in Fig. 5.1, and the dimensions are indicated in Table 5.1. Their diameters span from cm to microns and the velocities from m/s to less than mm/s in the capillaries. As illustrated, the vessel constantly curves and branches and repeatedly changes dimensions [44]. The blood flow is pulsatile, and the velocity rapidly changes both magnitude and direction as can be seen from the large difference between peak and mean velocities. In addition the peak Reynolds number is often above 2000, which indicates disturbed or turbulent flow in parts of the cardiac cycle.

All of these factors should be considered when devising a system for measuring blood the velocity in the human circulation. Foremost it must be a fast measurement system with 1 – 20 ms temporal resolution to follow the changes in velocity due to the acceleration from the pulsation. It should also have sub-millimeter spatial resolution in order to see changes in flow over space and visualize small vessels. A real-time system is also beneficial, since the flow patterns change rapidly, and it must be possible to quickly find the sites of flow. The system should ideally also be capable of finding and visualizing flow in all directions, since turbulent flow and vortices exists throughout the human circulation. Modern medical ultrasound systems can fulfill many of these demands, and the remaining part of this Chapter describes the physics and signal processing needed to perform velocity estimation along the ultrasound beam and in other directions.

5.3 Ultrasound scattering from blood

The constituents of blood are shown in Table 5.2. The most prominent part is the erythrocytes (red blood cells), where a mm^3 of blood contains roughly 5 million cells. The resolution of ultrasound systems is at best on the order of the cube of the wavelength λ^3, which is given by $\lambda = c/f_0$, where c is the speed of sound and f_0 the center frequency. For a 6 MHz system $\lambda = 0.26$ mm, which is much larger than a single cell. An ultrasound system, thus, only observes a large, random collection of cells.

The scattering from a single point scatterer or small cell can be described by the differential scattering cross section, σ_d. It is defined as the power scattered per unit solid angle at some angle Θ_s divided by the incident intensity [11, 51]:

$$\sigma_d(\Theta_s) = \frac{V_e^2 \pi^2}{\lambda^4} \left[\frac{\kappa_e - \kappa_0}{\kappa_0} + \frac{\rho_e - \rho_0}{\rho_e} \cos\Theta_s \right]^2, \tag{5.1}$$

where V_e is the volume of the scatterer, ρ_e is a small perturbation in density and κ_e in compressibility from their mean values ρ_0, κ_0. This results in a scattered field as shown in Fig. 5.2, where the scattering is dependent on the angle. It can, however, be seen that a signal is received in all directions.

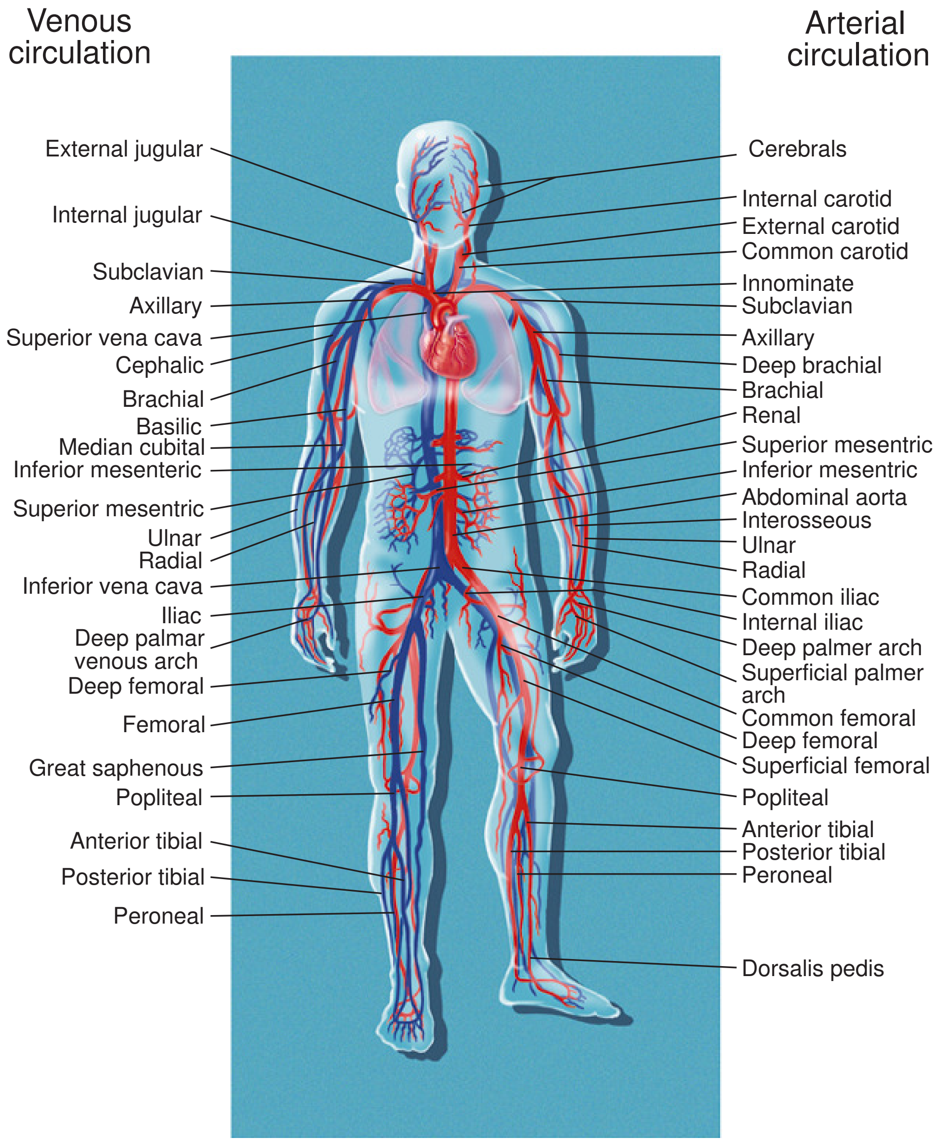

Figure 5.1: Major arteries and veins in the body (reprinted with permission from Abbott Laboratories, Illinois, USA).

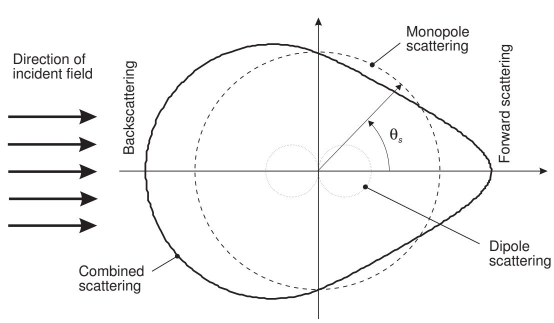

Figure 5.2: Monopole and dipole scattering from a perturbation in either compressibility or density (from [9]).

Table 5.1: Typical dimensions and flows of vessels in the human vascular system (data taken from [45, 46] and Table adapted from [9]).

Vessel	Internal diameter cm	Wall thickness cm	Length cm	Young's modulus $N/m^2 \cdot 10^5$
Ascending aorta	1.0 – 2.4	0.05 – 0.08	5	3 – 6
Descending aorta	0.8 – 1.8	0.05 – 0.08	20	3 – 6
Abdominal aorta	0.5 – 1.2	0.04 – 0.06	15	9 – 11
Femoral artery	0.2 – 0.8	0.02 – 0.06	10	9 – 12
Carotid artery	0.2 – 0.8	0.02 – 0.04	10 – 20	7 – 11
Arteriole	0.001 – 0.008	0.002	0.1 – 0.2	
Capillary	0.0004 – 0.0008	0.0001	0.02 – 0.1	
Inferior vena cava	0.6 – 1.5	0.01 – 0.02	20 – 40	0.4 – 1.0

Vessel	Peak velocity cm/s	Mean velocity cm/s	Reynolds number (peak)	Pulse propagation velocity cm/s
Ascending aorta	20 – 290	10 – 40	4500	400 – 600
Descending aorta	25 – 250	10 – 40	3400	400 – 600
Abdominal aorta	50 – 60	8 – 20	1250	700 – 600
Femoral artery	100 – 120	10 – 15	1000	800 – 1030
Common Carotid artery (range)	68 – 171	19-59		600 – 1100
Common Carotid artery (mean)	108	39		600 – 1100
Arteriole	0.5 – 1.0		0.09	
Capillary	0.02 – 0.17		0.001	
Inferior vena cava	15 – 40		700	100 – 700

	Mass density g/cm^3	Adiabatic compressibility 10^{-12} cm/dyne	Size μm	Particles per mm^3
Erythrocytes	1.092	34.1	2×7	$5 \cdot 10^6$
Leukocytes	-	-	9 – 25	$8 \cdot 10^3$
Platelets	-	-	2 – 4	$250 - 500 \cdot 10^3$
Plasma	1.021	40.9	-	-
0.9% saline	1.005	44.3	-	-

Table 5.2: Properties of the main components of blood. Data from [47, 48, 49, 50] and Table adapted from [9].

The signal received by the ultrasound transducer is therefore independent of the orientation of the vessel and it can be modeled as a random, Gaussian signal, as it emanates from a large collection of independent random scatterers. This give rise to the speckle pattern seen in ultrasound images. The scattering is very weak, as the density and compressibility perturbations are small compared to the surrounding tissue, and often the signal from blood is 20-40 dB lower than from the surrounding tissue. Vessels in an ultrasound image therefore appear black.

It should be noted that the signal received is random, but the same signal is received, if the experiment is repeated for the exact same collection of scatterers. This is a very important feature of ultrasound blood velocity signals and is heavily used in these systems, as will be clear in the following Sections. A small motion of the scatterers will therefore yield a second signal highly correlated with the first signal, when the motion $|v|T_{prf}$ is small compared to the beam width, and the velocity gradient is small across the scatterer collection observed. Here $|v|$ is velocity magnitude and T_{prf} is the time between the measurements.

5.4 Ultrasound signals from flowing blood

The derivation of velocity estimation methods is based on a model of ultrasound interaction between the moving blood and the ultrasound pulse. The model has to capture all the main features of the real received signal without unnecessary features and complications. This section will present a simple model for this interaction, which can readily be used for deriving new estimators and also give basic explanations for how to optimize the techniques. It can also be further expanded to the two- and three-dimensional velocity estimation methods presented in Sections 5.9 and 5.10.

The scattering from blood emanates from a large collection of independent, small point scatterers as described in Section 5.3. The one-dimensional received signal $y(t)$ can therefore be modeled as:

$$y(t) = p(t) * s(t), \tag{5.2}$$

which is the convolution of the basic ultrasound pulse $p(t)$ with the Gaussian, random scattering signal $s(t)$. The ultrasound pulse for flow estimation consists of a number of sinusoidal oscillations ($M_p = 4 - 8$) at the transducer's center frequency, f_0, convolved with the electro-mechanical impulse response of the transducer (from excitation voltage to pressure and from pressure to received voltage) [52]. Modern transducers are so broad band that a simple approximation is given by

$$p(t) = g(t)\sin(2\pi f_0 t), \tag{5.3}$$

where the envelope $g(t)$ is one from $t = 0$ to M_p/f_0 and zero elsewhere.

For a single point scatterer, the scattering can be modeled as a δ-function and the received signal is

$$y_1(t) = p(t) * a\delta\left(t - \frac{2d}{c}\right) = ap\left(t - \frac{2d}{c}\right), \tag{5.4}$$

where d is the distance to the point, c is the speed of sound (1540 m/s in tissue), and a is the scattering amplitude. There is, thus, a propagation delay $2d/c$ between transmitting the signal and receiving the response. For a moving scatterer this response will change as the scatterer moves farther away from the transducer. For a scatterer velocity of $v_z = |v|\cos\theta$ in the axial direction, and a time between measurements of T_{prf}, the second received signal is

$$y_2(t) = p(t) * a\delta\left(t - \frac{2d}{c} - \frac{2v_z T_{prf}}{c}\right) = ap\left(t - \frac{2d}{c} - \frac{2v_z T_{prf}}{c}\right). \tag{5.5}$$

Note that $v_z T_{prf} = T_{prf}|v|\cos\theta$ is the distance the scatterer moved along the ultrasound beam direction, where θ is the beam-to-flow angle. There is therefore an added delay of

$$t_s = \frac{2v_z}{c}T_{prf} \tag{5.6}$$

before the second signal is received. This is directly proportional to the velocity of the moving scatterer, and by determining this delay the axial velocity can directly be calculated. This can be achieved either through the phase shift estimator described in Section 5.7 or the time shift estimator in Section 5.8. Both estimators essentially just compare or correlate two received signals and find the shift in position between the received responses.

Combining (5.3) and (5.5) gives a model for the received signal for a number of pulse emissions [53, 54]:

$$y(t,i) = ap(t - \frac{2d}{c} - i\frac{2v_z}{c}T_{prf}) = ag(t - \frac{2d}{c} - it_s)\sin(2\pi f_0(t - \frac{2d}{c} - it_s)). \tag{5.7}$$

Here i is the pulse emission number. For pulsed systems the received signal is measured at a single depth corresponding to a fixed time t_x relative to the pulse emission. This is performed by taking out one sample from each pulse emission to create the sampled signal. To simplify things it can assumed that the pulse is long enough and the motion slow enough, so that the pulse amplitude stays roughly constant during the observation time. The sampled signal can then simply be written as

$$\begin{aligned} y(t_x, i) &= a\sin(2\pi f_0(t_x - \frac{2d}{c} - it_s)) \\ &= -a\sin(2\pi\frac{2v_z}{c}f_0 iT_{prf} - \phi_x). \end{aligned} \tag{5.8}$$

Here $\phi_x = 2\pi f_0(\frac{2d}{c} - t_x)$ is a fixed phase shift depending on the measurement depth. The sampling interval is T_{prf} and the frequency of the sampled signal is

$$f_p = \frac{2v_z}{c}f_0, \tag{5.9}$$

which is directly proportional to the velocity. Thus, estimating the frequency of this sampled signal $y(t_x, i)$ can directly reveal the axial velocity as described in the Section 5.6 on spectral velocity estimation. Often it is advantageous to also sample the signal in the depth direction, and this gives

$$y_s(n,i) = ag(n\Delta T - \frac{2d}{c} - it_s)\sin(2\pi f_0(n\Delta T - \frac{2d}{c} - it_s)), \tag{5.10}$$

which is often employed for averaging along the depth direction. Here $\Delta T = 1/f_s$ is the sampling interval and f_s the sampling frequency.

An illustration of the sampling process for a single moving scatterer is shown in Fig. 5.3, where the individual received signals are shown on the left graph. The scatterer is moving away from the transducer, and the time between emission and reception of the signal increases. A single sample is taken out for each received signal at the position of the red line. The resulting sampled signal is shown on the right graph, where the basic emitted pulse can be recognized. A low velocity will yield a long pulse and hence a low frequency of the received signal as indicated by (5.9). A high velocity will compress the signal and thereby give a high frequency. The velocity can be found by three different methods. A Fourier transform can be applied on the signal in the right graph to find the frequency as described in Section 5.6. The phase shift can be determined between the received signals. This is described in Section 5.7. The time shift can be found by correlating consecutive received signals, *i.e.* cross-correlating received signals in the left graph as described in Section 5.8.

The signal model can also be expanded to include a collection of scatterers moving at the same velocity

$$y(t,i) = p(t) * s(t - it_s - iT_{prf}) = p(t - it_s - iT_{prf}) * s(t) = p(t - it_s) * s(t). \tag{5.11}$$

as the same pulse is emitted every time. Using superposition this can just be seen as a summation of individual scatterers moving at their velocity and the model and observations made above can readily be applied.

5.4.1 Is it the Doppler effect?

The ultrasound systems for measuring velocity are often called Doppler systems implying that they rely on the Doppler shift to find the velocity. It is debatable whether this is really true. The Doppler effect can be described as a frequency shift on the received signal. The received signal can be written as:

$$y_D(t) = a\sin(2\pi(1 + \frac{2v_z}{c})f_0 t) \tag{5.12}$$

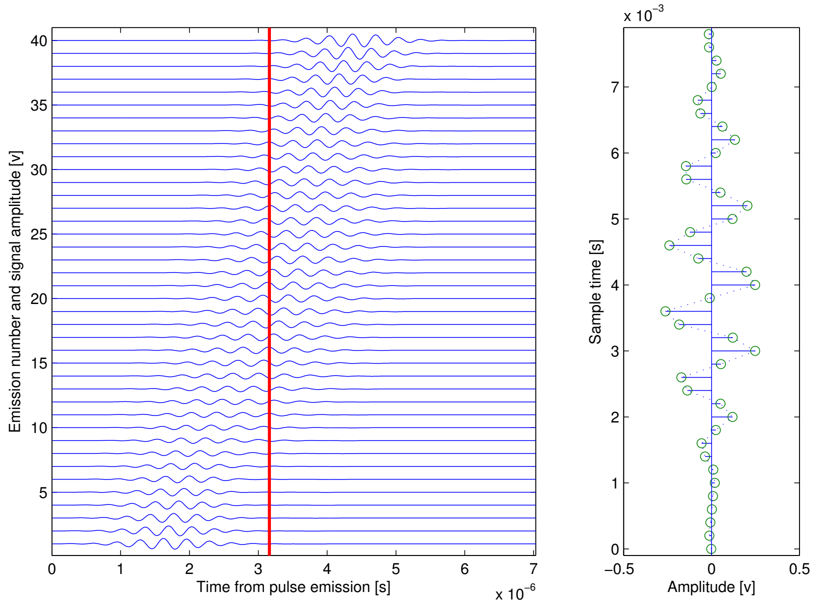

Figure 5.3: Received signals from a single scatterer moving away from the transducer. The solid red line in the left graph indicates the sampling instance for the samples show on the right graph.

for a simple continuous wave source. Here the received frequency is scaled by $(1+2v_z/c)$. There is, thus, a frequency difference of $f_D = \frac{2v_z}{c} f_0$ between the emitted and received signal. This is easy to determine for a continuous wave (CW) source and is used in CW systems, but it is extremely difficult for a pulsed source. Primarily, the shift in frequency is very small, on the order of 1/1000 of the emitted frequency. The shift is therefore minute compared to the bandwidth of the signal, which is roughly $B = f_0/M_p$, M_p being the number of cycles emitted. Secondly, finding this frequency shift assumes that the signal is not affected by other physical effects that shift the mean frequency of the received signal. This is not true in medical ultrasound as the Rayleigh scattering from blood is frequency dependent, and the dispersive attenuation in tissue is quite strong. Even for moderate tissue depths it can span from ten to hundreds of kilohertz of shift in the mean frequency of the received signal, thus, obscuring the Doppler shift [9]. This shift will be different and unknown from patient to patient and would therefore greatly influence the accuracy of the Doppler shift estimation. The Doppler effect is therefore not a reliable estimator of velocity and no pulsed system relies on this effect. Consequently it was not included in the model given in (5.10). Using the time or phase shift between consecutively received signal is a more reliable method, which is not affected much by attenuation, scattering, beam modulation, or non-linear propagation that all affect the received ultrasound signal [9]. The reason for this is that two signals are compared (correlated) and only the difference between two emissions is used in the velocity estimation. This results in a much more reliable system, which does not require an extremely precise frequency content to work. I, therefore, prefer not to call these system Doppler systems as it is confusing to what effect is actually used for the velocity estimation.

5.5 Simulation of flow signals

The model presented above can also be used for simulating the signal from flowing blood. The Field II simulation system [55, 38] uses a model based on spatial impulse responses. The received voltage signal is here described by [6]:

$$v_r(t, \vec{r}_1) = v_{pe}(t) \underset{t}{\star} f_m(\vec{r}_2) \underset{r}{\star} h_{pe}(\vec{r}_1, \vec{r}_2, t), \tag{5.13}$$

where f_m accounts for the scattering by the medium, h_{pe} describes the spatial distribution of the ultrasound field, and v_{pe} is the one-dimensional pulse emanating from the pulse excitation and conversion from voltage to pressure and back again. $\vec{r}_1$ is the position of the transducer and $\vec{r}_2$ is the position of the scatterers. $\underset{t}{\star}$ and $\underset{r}{\star}$ denote temporal and spatial convolution. This equation describes a summation of the responses from all the point scatterers properly weighted by the ultrasound field strength as described by h_{pe} and convolved with v_{pe}. This yields the signal for a collection of scatterers when the Doppler effect is neglected. For the next emission the scatterer's positions should be propagated as:

$$\vec{r}_2(i+1) = \vec{r}_2(i) + T_{prf}\vec{v}(\vec{r}_2(i), iT_{prf}) \tag{5.14}$$

where $\vec{v}(\vec{r}_2(i), t)$ denotes the velocity vector for this point scatterer for emission i at time $t = iT_{prf}$. Propagating the scatterers and calculating the received signal will then yield a realistic flow signal, which is usable for developing, validating, and evaluating pulsed ultrasound flow systems.

A typical parabolic velocity profile for stationary, laminar flow is:

$$v(r) = \left(1 - \frac{r^2}{R^2}\right) v_0, \tag{5.15}$$

where R is the vessel radius and v_0 is the peak velocity in the vessel. More realistic velocity profiles can be generated by using the Womersley-Evans' description of pulsatile flow [56, 57]. Here a few parameters can be used to describe the full temporal and spatial evolution of the pulsatile flow in *e.g.* the carotid or femoral arteries, and this can readily be included in the simulation. It is also possible to combine this method with computational fluid dynamics using finite element modeling for the flow [58] to capture the formation of turbulence and vortices.

5.6 Estimation of the velocity distribution

The frequency of the flow signals measured is directly proportional to the blood velocity as shown in (5.9). Finding the frequency content of the signal therefore reveals the velocity distribution in the vessel under investigation. This is utilized in spectral estimation systems, which often combine the measurement of velocity with an anatomic B-mode image as shown in Fig. 5.4. The top image shows the anatomy, and the measurement range gate for the flow is indicated by the broken line. The flow measurement is conducted within the vessel, and the "wings" indicate the assumed flow direction. The bottom display shows the velocity distribution as a function of time for five heart beats in the carotid artery.

The velocity distribution changes over the cardiac cycle due to the pulsation of the flow, and the frequency content of the received signal is, thus, not constant. The direction of the flow can also be seen here. For the carotid artery the spectrum is one-sided (only positive frequency components) as the flow is uni-directional towards the brain. It is, thus, important to have processing that can differentiate between velocities towards or away from the transducer. This can be achived by using complex signals with a one sided spectrum. Making a Hilbert transform on the received Radio Frequency (RF) signal and forming the analytic signal [59] then gives

$$r_s(n_x, i) = ag_s(i)\exp(j(2\pi\frac{2v_z}{c}f_0 iT_{prf} - \phi_x)). \tag{5.16}$$

Here the emitted frequency f_0 is scaled by $\frac{2v_z}{c}$, which can be positive or negative depending on the sign of the velocity. Making a Fourier transform of $r_s(n_x, i)$ along the i direction will yield a one-sided spectrum, so that the velocity direction can be determined.

The signal from a vessel consists of a weighted superposition from all the primarily red blood cell scatterers in the vessel. They each flow at slightly different velocities, and calculating the power density spectrum of the signal will give the corresponding velocity distribution of the cells [60, 9]. Displaying the spectrum therefore visualizes the velocity distribution. This has to be shown as a function of time to reveal the dynamic changes in the spectrogram.

Modern ultrasound scanners employ a short time Fourier transform. The complex signal is divided into segments of typically 128 to 256 samples weighted by *e.g.* a von Hann window before Fourier transformation. The process is repeated every 1-5 ms as the spectra are displayed side-by-side as a function of time as shown in Fig. 5.4. Compensating for the beam-to-flow angle then gives a quantitative and real-time display of the velocity distribution at one given position in the vessel.

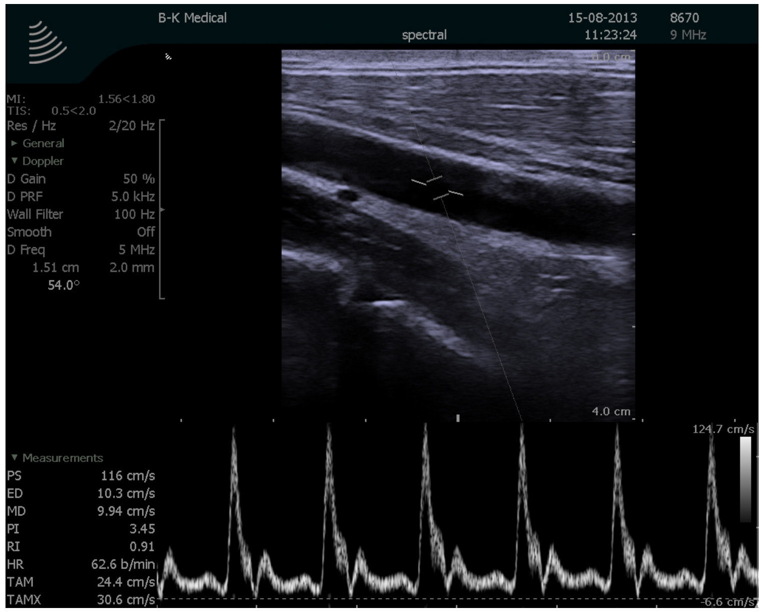

Figure 5.4: Duplex mode ultrasound imaging showing the anatomic B-mode image on the top and the spectral velocity distribution on the bottom (Image courtesy of MD Peter M. Hansen).

The range gate can be selected to be small or large depending on whether the peak velocity or the mean velocity is investigated. The averaging over the range gate can be made either by selecting the length of the emitted pulse or by averaging the spectra across the depth direction by calculating one spectrum for each depth sample n. The spectrogram acquisition method is used clinically when quantitative parameters like peak velocity, mean velocity, or resistive index for the flow must be calculated.

5.7 Axial velocity estimation using the phase

Spectral systems only display the velocity distribution at one single position in the vessel. Often it is preferred to visualize the velocity in a region using the so called Color Flow Mapping (CFM) method. Here data are acquired in a number of directions to construct a real time image of the velocity [61]. The method gives a single value for the velocity at each spatial position, but only a very limited amount of data is available to maintain a reasonable frame rate. Often 8-16 emissions are made in the same direction, and the velocity is found in this direction as a function of depth. The acquisition is then repeated in other directions, and an image of velocity is made and superimposed on the normal B-mode image as shown in Fig. 5.5.

These systems find the velocity from the phase shift between the acquired lines. The complex received signal is in continuous time written as

$$r_t(t) = a g_s(t) \exp(-j(2\pi \frac{2v_z}{c} f_0 t - \phi_x)). \tag{5.17}$$

Taking the derivate of its phase gives

$$\phi' = \frac{d\phi}{dt} = \frac{d(-2\pi \frac{2v_z}{c} f_0 t + \phi_x))}{dt} = -2\pi \frac{2v_z}{c} f_0, \tag{5.18}$$

so that the estimated velocity is

$$\hat{v}_z = -\frac{\phi'}{4\pi f_0} c. \tag{5.19}$$

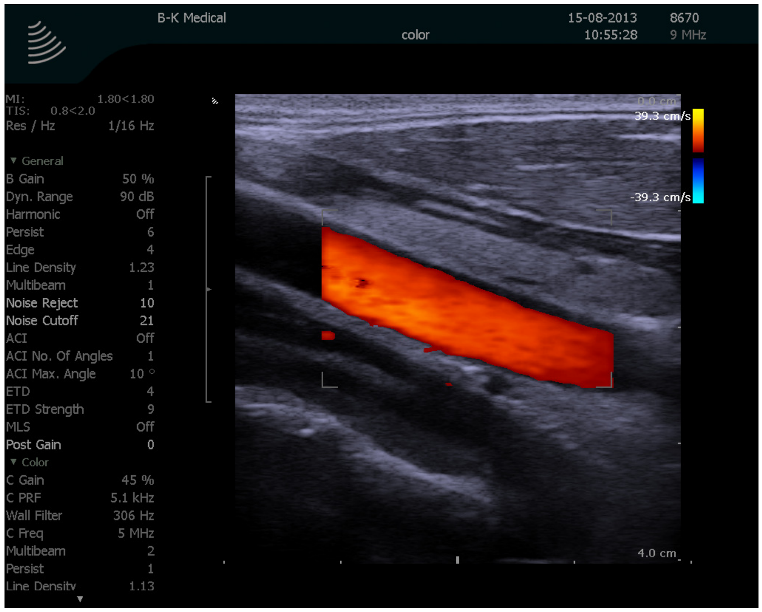

Figure 5.5: CFM image of the carotid artery. The red colors indicate velocity towards the transducer and blue away from the transducer (Image courtesy of MD Peter M. Hansen).

The discrete version can be written as

$$r_s(n_x, i) = a g_s(i) \exp(-j(2\pi \frac{2v_z}{c} f_0 i T_{prf} - \phi_x)) = x(i) + jy(i). \tag{5.20}$$

The phase difference can be found from

$$\begin{aligned} \Delta\phi &= \phi(i+1) - \phi(i) = \arctan \frac{y(i+1)}{x(i+1)} - \arctan \frac{y(i)}{x(i)} \\ &= -(2\pi \frac{2v_z}{c} f_0 (i+1) T_{prf} - \phi_x) + (2\pi \frac{2v_z}{c} f_0 i T_{prf} - \phi_x) \\ &= -2\pi \frac{2v_z}{c} f_0 T_{prf} \end{aligned} \tag{5.21}$$

and the velocity can be estimated from

$$\hat{v}_z = -\frac{\Delta\phi}{4\pi T_{prf} f_0} c = \frac{\Delta\phi}{2\pi} \frac{f_{prf}}{2f_0} c = -\frac{\Delta\phi}{2\pi} \frac{f_{prf}}{2} \lambda = -\frac{\Delta\phi}{4\pi} \frac{\lambda}{T_{prf}}. \tag{5.22}$$

The maximum unique phase difference that can be estimated is $\Delta\phi = \pm\pi$, and the largest unique velocity is therefore $\hat{v}_{z,max} = \frac{\lambda}{4T_{prf}}$. This is determined by the wavelength used and the pulse repetition time or essentially sampling interval. The pulse repetition time of course has to be sufficiently large to cover the full depth and must be larger than $T_{prf} > 2d/c$. Combining the two limitations gives the depth-velocity limitation:

$$\hat{v}_{z,max} < \frac{c}{4} \frac{\lambda}{2d} = \frac{c}{8 d f_0} c, \tag{5.23}$$

which limits the maximum detectable velocity for a given depth, and is a limitation imposed by the use of a phase estimation system.

The velocity estimation is not performed by taking the phase difference between two measurements, but rather by combing the two arctan operations from:

$$\tan(\Delta\phi) = \tan\left(\arctan\left(\frac{y(i+1)}{x(i+1)}\right) - \arctan\left(\frac{y(i)}{x(i)}\right)\right)$$

$$= \frac{\frac{y(i+1)}{x(i+1)} - \frac{y(i)}{x(i)}}{1 + \frac{y(i+1)}{x(i+1)} \frac{y(i)}{x(i)}}$$

$$= \frac{y(i+)x(i) - y(i)x(i+1)}{x(i+1)x(i) + y(i+1)y(i)} \tag{5.24}$$

using that

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}. \tag{5.25}$$

Then

$$\arctan\left(\frac{y(i+1)x(i) - y(i)x(i+1)}{x(i+1)x(i) + y(i+1)y(i)}\right) = -2\pi f_0 \frac{2v_z}{c} T_{prf} \tag{5.26}$$

or

$$\hat{v}_z = -c\frac{f_{prf}}{4\pi f_0} \arctan\left(\frac{y(i+1)x(i) - y(i)x(i+1)}{x(i+1)x(i) + y(i+1)y(i)}\right). \tag{5.27}$$

This simple algebraic equation directly yields the velocity from comparing two emissions. Often the signal-to-noise ratio from blood signals is low due to the weak scattering from blood, and therefore it is advantageous to average over a number of emissions as

$$\hat{v}_z = -c\frac{f_{prf}}{4\pi f_0} \arctan\left(\frac{\sum_{i=1}^{M} y(i+1)x(i) - y(i)x(i+1)}{\sum_{i=1}^{M} x(i+1)x(i) + y(i+1)y(i)}\right), \tag{5.28}$$

where M is the number of emissions. This can also be calculated as the phase of the lag one autocorrelation of the received signal [61]. Further averaging can be made along the depth direction as the data is highly correlated over a pulse length. This is calculated as [62]:

$$\hat{v}_z(N_x) = -c\frac{f_{prf}}{4\pi f_0} \arctan\left(\frac{\sum_{i=1}^{M}\sum_{n=-N_p/2}^{N_p/2} y(n+N_x,i+1)x(n+N_x,i) - y(n+N_x,i)x(n+N_x,i+1)}{\sum_{i=1}^{M}\sum_{n=-N_p/2}^{N_p/2} x(n+N_x,i+1)x(n+N_x,i) + y(n+N_x,i+1)y(n+N_x,i)}\right) \tag{5.29}$$

when the real part of received data is given as $x(n, i)$ and $y(n, i)$ is the imaginary part. Here n is the time index (depth) and i is the pulse emission number. N_x is the starting sample for the depth to estimate the velocity. N_p is the number of RF samples to average over and typically corresponds to one pulse length. This is the approach suggested by Loupas et al. [62] and is the one used in nearly all modern scanners.

5.7.1 Stationary echo canceling

At vessel boundaries the received signal consists of both reflections from the vessel boundary and the scattered signal from blood. Often the reflection signal is 20 to 40 dB larger in amplitude compared with the signal from blood, and it therefore makes velocity estimation heavily biased or impossible. The reflection signal is often assumed to be stationary and thereby constant over the number of pulse emissions. Subtracting two consecutive signals will therefore remove the stationary component and leave a signal suitable for velocity estimation:

$$r_{es}(n, i) = r_s(n, i) - r_s(n, i+1) \tag{5.30}$$

For a fully stationary signal this will give zero whereas the flow signal will be filtered depending on the correlation between the two emissions. This filtration on the flow part $r_f(n, i)$ of the signal can be calculated from

$$r_e(n, i) = r_f(n, i) - r_f(n, i+1) = r_f(n, i) - r_f(n, i)\exp(j2\pi\frac{2v_z}{c} f_0 T_{prf}), \tag{5.31}$$

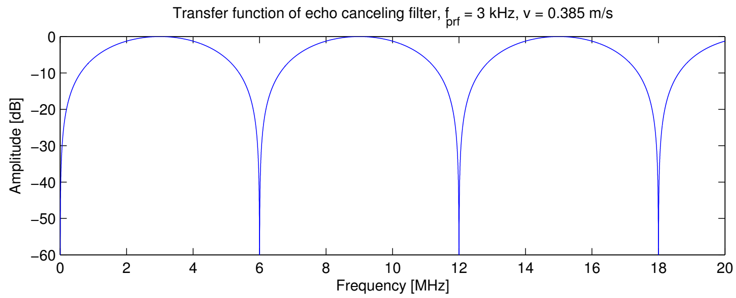

Figure 5.6: Transfer function of echo canceling filter.

assuming that the signals are so highly correlated that the shift in position due to the flow can be described by a simple phase shift. In the Fourier domain this gives

$$R_e(f) = R_f(f)(1 - \exp(j2\pi \frac{2v_z}{c} f T_{prf})), \tag{5.32}$$

where the Fourier transform is taken along the emissions (i). The transfer function of the filter is therefore

$$H(f) = (1 - \exp(j2\pi \frac{2v_z}{c} f T_{prf})) = 2j \sin(\pi \frac{2v_z}{c} T_{prf} f). \tag{5.33}$$

The transfer function of this filter is shown in Fig. 5.6 for $v_z = 0.385$ m/s and $f_{prf} = 3$ kHz. It can be seen that the filter reduces the energy of the RF signal significantly around a band of 6 MHz and at low frequencies. It is an unavoidable side-effect of echo canceling filtration that the energy of the flow signal is reduced and the signal-to-noise ratio is therefore reduced. The effect is especially noticeable at low flow velocities. Here the second signal is very similar to the first measurement since the time shift $t_s = 2v_z/cT_{prf}$ is small. The subtraction therefore removes most of the energy and the noise power in the two measurements are added.

The reduction in signal-to-noise ratio due to the stationary echo canceling can be analytically calculated for a Gaussian pulse and is [9, 63]:

$$\begin{aligned}
R_{\text{snr}} &= \sqrt{\frac{2\sqrt{2} + \exp(-\frac{2}{B_r^2})}{2\sqrt{2} + \exp(-\frac{2}{B_r^2})\xi_1 - 2\sqrt{2}\xi_2 \cos(2\pi \frac{f_0}{f_{sh}})}} \\
\xi_1 &= 1 - \exp\left(-\frac{1}{2}\left(\frac{\pi B_r f_0}{f_{sh}}\right)^2\right) \\
\xi_2 &= \exp\left(-\left(\frac{\pi B_r f_0}{f_{sh}}\right)^2\right) \\
f_{sh} &= \frac{c}{2v_z} f_{prf},
\end{aligned} \tag{5.34}$$

where the pulse is given by

$$p(t) = \exp(-2(B_r f_0 \pi)^2 t^2) \cos(2\pi f_0 t). \tag{5.35}$$

Here B_r is the relative bandwidth and f_0 is the center frequency. The reduction for $f_0 = 3$ MHz, $f_{prf} = 3$ kHz and $B_r = 0.08$ is shown in Fig. 5.7. At zero velocity the decrease is infinite as the two signals are identical and no velocity can be found. The reduction decreases progressively for increasing velocity and a gain in SNR is found at the maximum detectable velocity. Here the two signals are inverted compared to each other and the subtraction then yields an addition of the two. The amplitude is therefore doubled and the noise power is doubled, hence giving an improvement of 3 dB in SNR. The curve in Fig. 5.7 is dependent on the echo canceling filter used and on the pulse emitted $p(t)$, but there will also be an infinite loss at zero velocity.

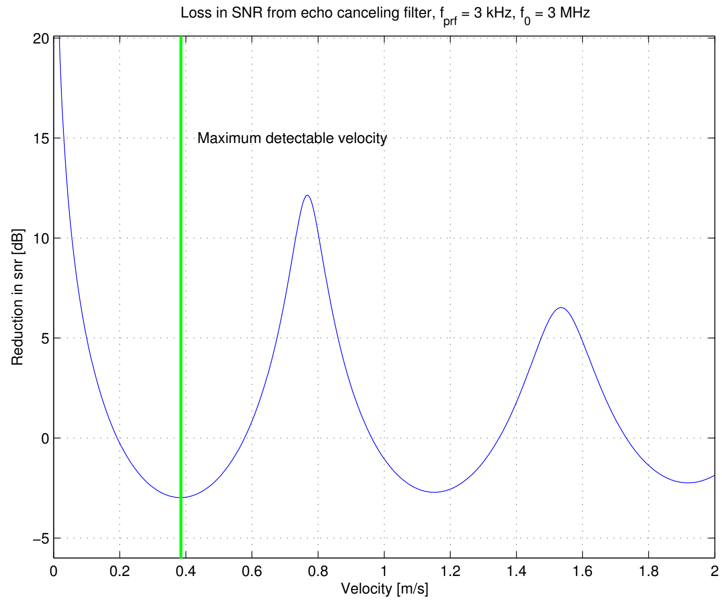

Figure 5.7: Loss in signal-to-noise ratio due to the echo canceling filter.

Another method is to subtract the mean value of all the received lines as

$$r_e(n,i) = r_s(n,i) - \frac{1}{M}\sum_{k=1}^{M} r_s(n,k). \tag{5.36}$$

This gives a sharper cut-off in transfer function of the filter and less noise added to the response.

Many different echo canceling filters have been suggested [64, 65], but it still remains a very challenging part of velocity estimation and probably the one factor most affecting the outcome of the estimation.

5.8 Axial velocity estimation using the time shift

It is also possible to estimate the velocity directly from the time shift between the signals. Two consecutive signals are related by:

$$y_c(t, i+1) = y_c(t - \frac{2v_z}{c}T_{prf}, i) = y_c(t - t_s, i) \tag{5.37}$$

Cross-correlating two consecutive signals can then be used for finding the time shift and, hence, the velocity. This is calculated by [54, 53]:

$$R_{12}(\tau, i) = \int_T y_c(t,i)y_c(t+\tau, i+1)dt = \int_T y_c(t,i)y_c(t - t_s + \tau, i)dt = R_{11}(\tau - t_s) \tag{5.38}$$

Using (5.2) the autocorrelation can be rewritten as

$$\begin{aligned} R_{11}(\tau - t_s, i) &= \int_T p(t) * s(t,i)p(t) * s(t - t_s + \tau, i)dt = R_{pp}(\tau) * \int_T s(t,i)s(t - t_s + \tau, i)dt \\ &= R_{pp}(\tau)P_s\delta(\tau - t_s) = P_s R_{pp}(\tau - t_s), \end{aligned} \tag{5.39}$$

where P_s is the scattering power, and the scattering signal is assumed to be random and white. $R_{pp}(\tau)$ is the autocorrelation of the emitted pulse, and this has a unique maximum value at $\tau = 0$. $R_{pp}(\tau - t_s)$ therefore has a unique

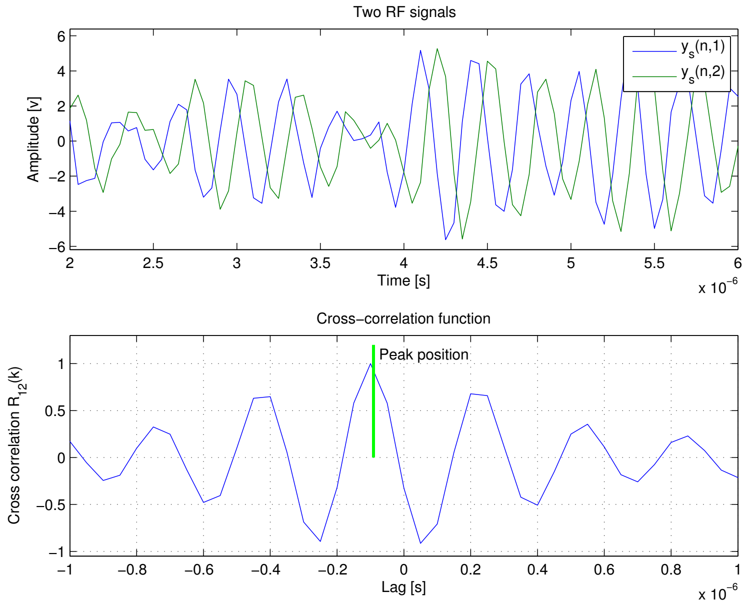

Figure 5.8: Illustration of the cross-correlation. The top graph shows the two signals to be correlated and the bottom graph shows their cross-correlation.

maximum at $\tau = t_s$, and the velocity can be found from

$$\hat{v}_z = c\frac{\hat{t}_s}{2T_{prf}}. \tag{5.40}$$

The cross-correlation is calculated from the discrete sampled signals $y_s(n, i)$ as

$$R_{12d}(k, N_x) = \sum_{i=1}^{N_e-1} \sum_{n=-N_n/2}^{N_n/2} y_s(n + N_x, i) y_s(n + k + N_x, i + 1) \tag{5.41}$$

where N_e is the number of emissions to average over, N_n is the number of samples to average over, and N_x is the sample number (depth) at which to find the velocity at. The position of the peak n_s in $R_{12}(k, N_x)$ is then found at the velocity calculated from

$$\hat{v}_z = c\frac{\hat{n}_s/f_s}{2T_{prf}}, \tag{5.42}$$

where f_s is the RF sampling frequency.

The cross-correlation function is shown in Fig. 5.8. The upper graph shows the two signals used where the time shift readily can be seen. The lower graph shows the cross-correlation of the signals along with an indication of the peak position. A 3 MHz Gaussian pulse was used in the simulation along with $f_{prf} = 5$ kHz and a 20 MHz sampling frequency. The velocity was 0.35 m/s, which gave a time shift of $t_s = 0.156$ μs.

The time shift is usually comparable the sampling interval $1/f_s$, and the velocity estimates will be heavily quantized. This can be solved by fitting a second-order polynomial around the cross-correlation peak and then finding the peak value of the polynomial. The interpolation is calculated by [66]:

$$n_{int} = n_s - \frac{\hat{R}_{12d}(n_s + 1) - \hat{R}_{12d}(n_s - 1)}{2(\hat{R}_{12d}(n_s + 1) - 2\hat{R}_{12d}(n_s) + \hat{R}_{12d}(n_s - 1))} \tag{5.43}$$

and the interpolated estimate is given by

$$\hat{v}_{int} = \frac{c}{2}\frac{n_{int} f_{prf}}{f_s}. \tag{5.44}$$

This gives an increased resolution, if the cross-correlation estimate is sufficiently noise-free.

Several factors affect how well this estimator works. A short pulse should be used for the emission to provide the narrowest possible cross-correlation function. The averaging should by performed over a sufficient number of samples to give a good estimate, but should also be limited to a region where the velocity can be assumed to be constant in space. Also, the number of emissions averaged over time should be sufficient to ensure a good result without lowering the frame rate too much.

The cross-correlation method can find velocities higher than the autocorrelation approach as it is not restricted to a phase shift of $\pm\pi$. In theory any kind of velocity can be found, but this can lead to false peak detection [67]. Here a global maximum in the correlation function beyond the correct peak is found. As the cross-correlation is determined by the autocorrelation of the pulse, these erroneous peak will reside at $k/f_0 + n_s$, leading to a large error in the estimated velocity. This can be difficult to correct and gives spike artifacts in the image display. Often the search range for finding the maximum is therefore limited to lie around zero velocity. No false peaks arise if the search range is limited to $-f_s/(2f_0) < k < f_s/(2f_0)$ which gives the same maximum velocity as the autocorrelation or phase shift approach.

It is difficult to decide which of the two methods are best. Often the cross-correlation gives better accuracies on the estimates, but this is off-set by its lower sensitivty, which comes from using a shorter pulse. In general the decision is dependent on the actual measurement situation and setup.

5.9 Two-dimensional vector velocity estimation

The methods described so far only find the velocity along the ultrasound beam direction, and this is often perpendicular to the flow direction. The velocity component found is therefore often the smallest and least important. Many angle compensation schemes have been devised [68, 69] but they all rely on the assumption that a single angle applied for the whole cardiac cycle and region of interest, which in general is not correct. Due to the pulsating nature of the flow, the velocity will often be in all directions and changes both magnitude and direction over the cardiac cycle. There is, thus, a real need for vector velocity estimation methods.

The problem has been acknowledged for many years, and a number of authors have suggested schemes for finding the velocity vector. Fox [70] used two crossing beams to find the velocity for two directions and then combine it to yield the 2-D velocity vector. Newhouse et al. [71] suggested using the bandwidth of the received signal to determine the lateral component. Trahey et al. [72] used a speckle tracking approach for searching for the velocity vector.

Currently the only method that has been introduced on commercial FDA approved scanners is the Transverse Oscillation (TO) approach developed by Jensen and Munk [73, 74]. A similar approach was also suggested by [75].

The traditional axial velocity estimation methods rely on the axial oscillation to find the velocity. The TO method introduces an oscillation transverse to the ultrasound propagation direction to make the received signals sensitive to a transverse motion. Such a transverse field is shown in Fig. 5.9. The figure shows a contour plot of the linear point spread function (PSF), and oscillations can be seen both in the lateral and axial direction. Two fields are needed to make a complex field with an in-phase and quadrature component that can be used for finding the sign of the velocity in the lateral direction.

The lateral oscillation is generated by utilizing a special apodization on the transducer during receive processing. At the focus there is a Fourier relation between the transducer's apodization function and the ultrasound field [76]. To generate a sinusoidal oscillation, the receive apodization should ideally, derived for a continuous field, consist of two sinc shaped peaks with a distance of D. This will give a lateral wavelength of:

$$\lambda_x = \lambda \frac{2D}{P_d}, \tag{5.45}$$

where P_d is the depth in tissue. Sending out a fairly broad beam and focusing the two fields in receive with this apodization function will yield the fields shown in Fig. 5.9. The signals at the dashed line are shown in Fig. 5.10,

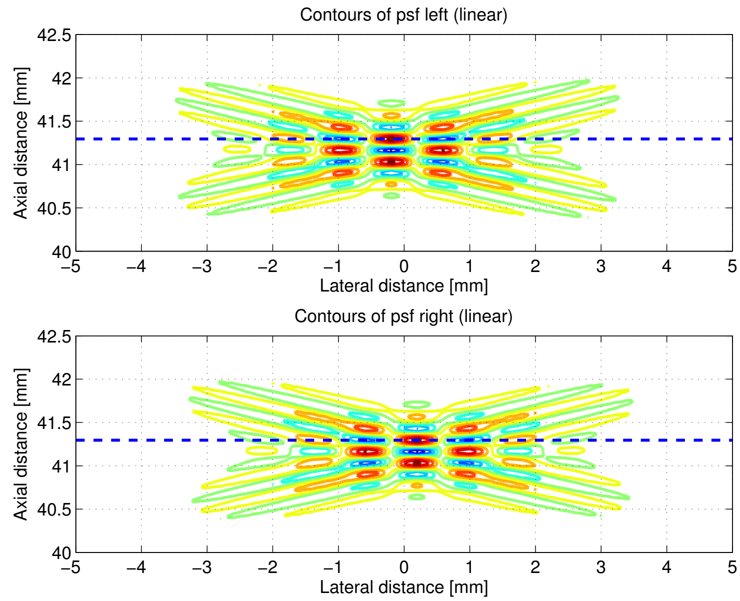

Figure 5.9: Ultrasound fields employed in TO velocity estimation. The top graph shows the left field and the bottom the right field.

where the blue curve is from the left field and the dashed green curve is from the right field. These two signals should ideally be 90° phase shifted compared to each other to generate a one-side spectrum. For a pulsed field (5.45) is not accurate enough to ensure this and computer optimization has been made to adjust the focusing to give the best possible result [77]. This has been done for a convex array probe by minimizing the amplitude spectrum of the complex field for negative spatial frequencies as shown in the lower graph in Fig. 5.10. This ensures a nearly one sided spectrum and thereby the best possible estimate in terms of bias and standard deviation.

The receive beamforming yields signals usable for the vector velocity estimation. They are combined into the complex signal $r_{sq}(i)$ and a temporal Hilbert transform of this gives the signal $r_{sqh}(i)$. From these two new signals are made:

$$\begin{aligned} r_1(i) &= r_{sq}(i) + j r_{sqh}(i) \\ r_2(i) &= r_{sq}(i) - j r_{sqh}(i). \end{aligned} \tag{5.46}$$

The velocity components are then estimated by the TO estimators derived in [74]. They are given by:

$$v_x = \frac{\lambda_x}{2\pi 2 T_{prf}} \arctan\left(\frac{\Im\{R_1(1)\}\Re\{R_2(1)\} + \Im\{R_2(1)\}\Re\{R_1(1)\}}{\Re\{R_1(1)\}\Re\{R_2(1)\} - \Im\{R_1(1)\}\Im\{R_2(1)\}}\right) \tag{5.47}$$

and

$$v_z = \frac{c}{2\pi 4 T_{prf} f_0} \arctan\left(\frac{\Im\{R_1(1)\}\Re\{R_2(1)\} - \Im\{R_2(1)\}\Re\{R_1(1)\}}{\Re\{R_1(1)\}\Re\{R_2(1)\} + \Im\{R_1(1)\}\Im\{R_2(1)\}}\right). \tag{5.48}$$

where $R_1(1)$ is the complex lag one autocorrelation value for $r_1(i)$, and $R_2(1)$ is the complex lag one autocorrelation value for $r_2(i)$.$\Im$ denotes imaginary par and $\Re$ the real part. These give the velocity vector in the imaging plane.

Fig. 5.11 shows a vector flow image (VFI) of the carotid bifurcation measured by a linear array probe and the TO approach. The image is acquired right after peak systole. The vectors show magnitude and direction of the flow, while the color intensities show velocity magnitude. A vortex can be seen in the carotid bulb. The vortex appears right after peak systole and disappears in roughly 100 ms. This is a normal flow pattern in humans and shows the value of vector flow imaging. It is important to note that there is no single correct beam-to-flow angle in this image. Both magnitude and direction change rapidly as a function of both time and space, making it essential to have a vector flow estimation system to capture the full complexity of the hemodynamics.

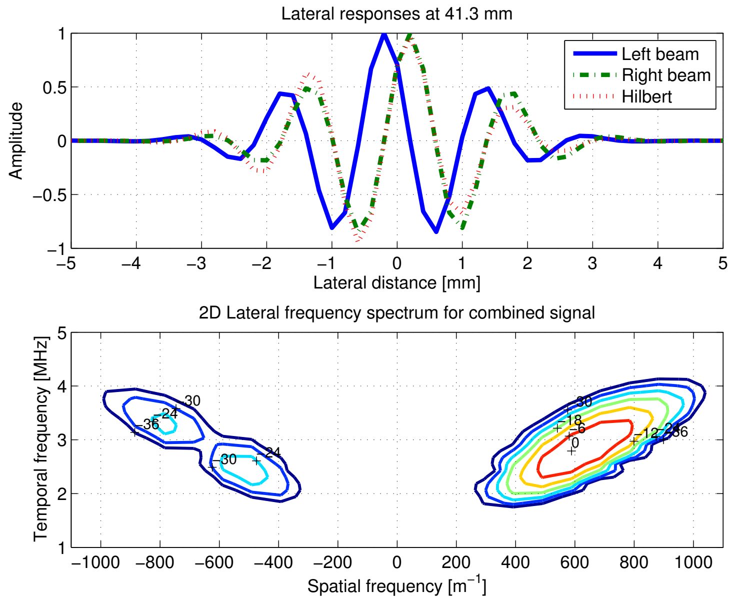

Figure 5.10: Lateral left and right responses in the TO fields at the maximum compared to the Hilbert transform of the left field. The bottom graph shows the 2-D Fourier transform of the complex TO psf.

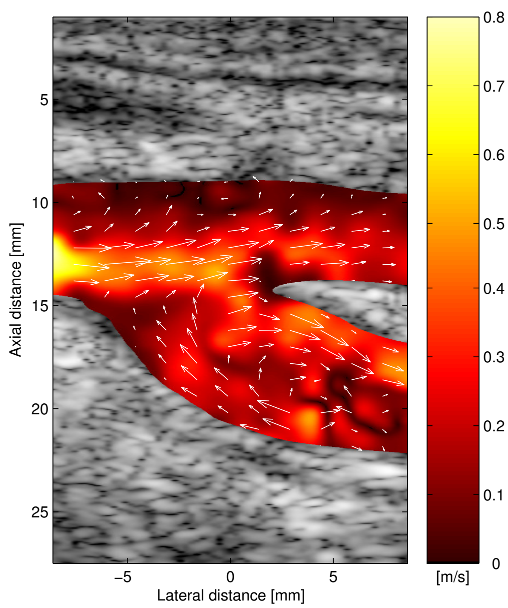

Figure 5.11: Vector flow image of the carotid bifurcation right after peak systole, where a vortex is present in the carotid bulb (from [78]).

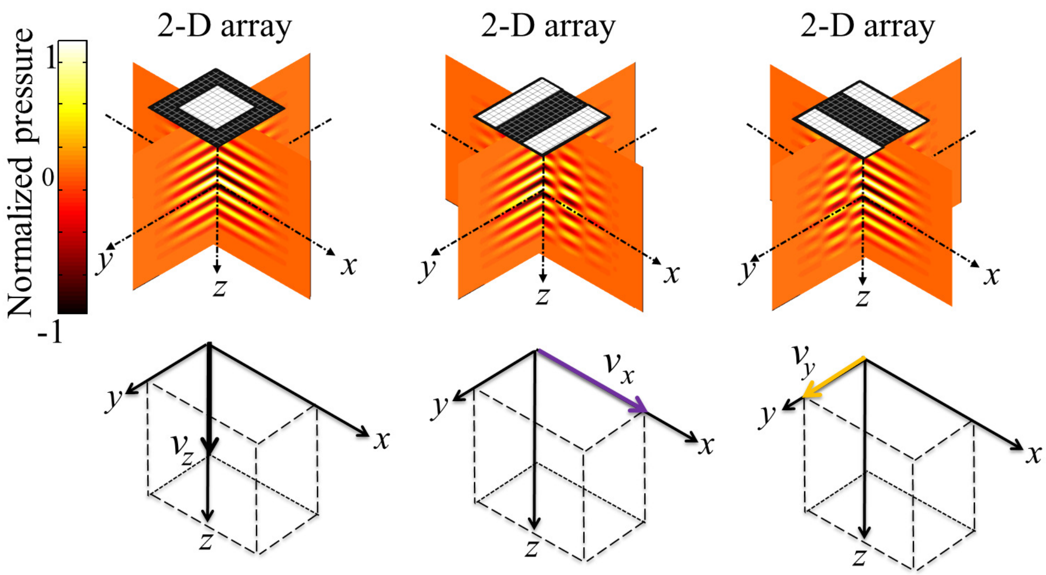

Figure 5.12: Receive apodization profiles applied to generate the TO fields for all three velocity components. The white shaded areas indicate the active elements in the 32 x 32 array (from [79]).

5.10 Three-dimensional vector velocity estimation

The TO approach can also be extended to full three-dimensional imaging by using a 2-D matrix transducer. Here 5 lines are beamformed in parallel during the receive processing to get a set of lines for the transverse, elevation, and axial velocity components. The beamforming is visualized in Fig. 5.12, where the active elements used in the beamformation are shaded white and the corresponding velocity direction is indicated below the array. Slices into the corresponding ultrasound fields are also shown below the transducer, and the oscillations in the axial, transverse, and elevation planes can be seen.

The approach has been implemented on the SARUS experimental ultrasound scanner [80] connected to a Vermon 32 x 32 3MHz matrix array transducer [81, 82, 79]. Parabolic flow in a recirculating flow rig was measured, and the result is shown in Fig. 5.13. The flow is in the elevation direction (out of the imaging plane) of the image shown in the bottom, and both 1-D and 2-D velocity estimation systems would show no velocity. The arrows indicate the out-of-plane motion amplitude and direction and show the parabolic velocity profile.

The approach has also been used *in-vivo* as shown in Fig. 5.14 for the carotid artery. Two intersecting B-mode images have been acquired, and the 3-D velocity vectors have been found at the intersection of the two planes. The estimated velocity magnitude as a function of time is shown in the lower graph, and the velocity vector is shown around the peak systole in the cardiac cycle. This method has the potential of showing the full dynamics of the complex flow in the human circulation in real time for a complete evaluation of the hemodynamics.

5.11 Synthetic aperture and plane wave flow estimation

The measurements systems described so far are all sequential in nature. They acquire the flow lines in one direction at a time, and this makes the measurement slow, especially when images consist of many directions or many emissions have to be used for flow estimation. Triplex imaging shows both the B-mode, CFM image, and spectral information simultaneously and therefore needs to split the acquisition time between the three modes. This often makes the resulting frame rate unacceptable low for clinical use for large depths. This will also be a very limiting factor for 3-D

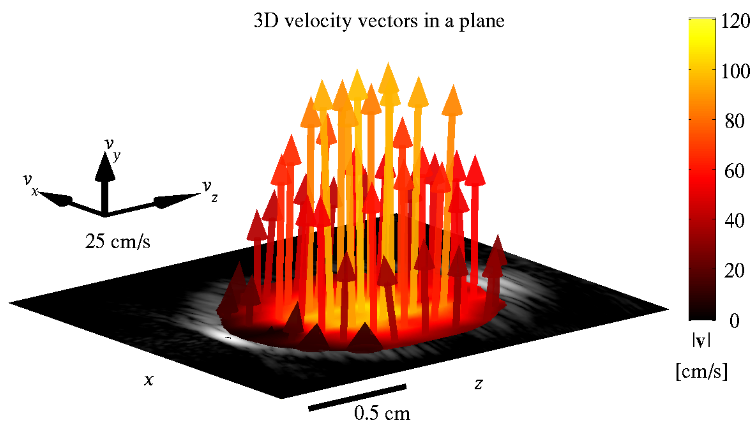

Figure 5.13: 3D vector velocity image for a parabolic, stationary flow(from [79]).

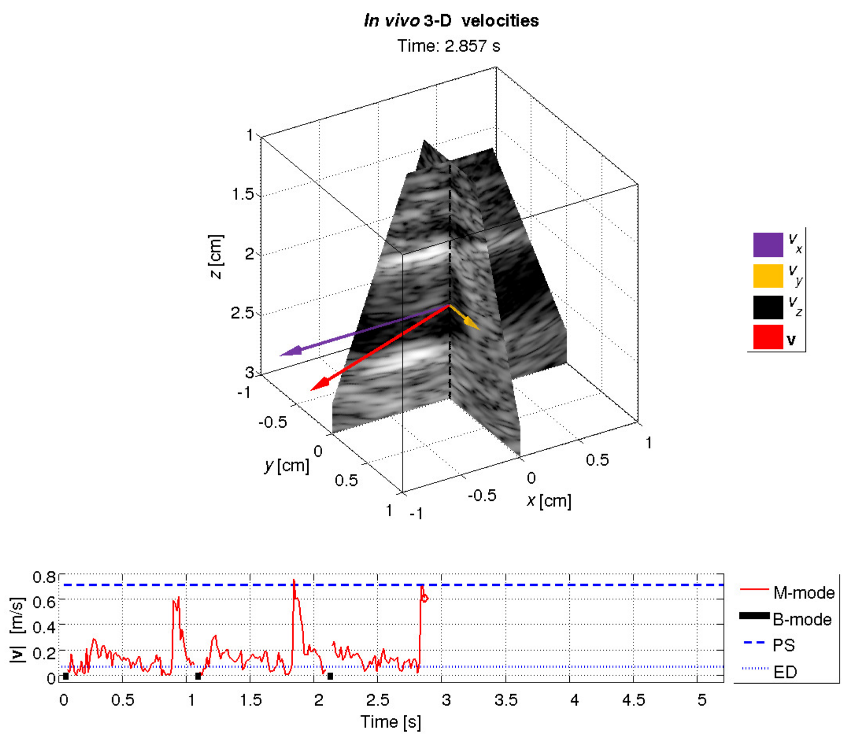

Figure 5.14: In-vivo 3-D vector velocity image taken around peak systole in the carotid artery of a healthy volunteer (Courtesy of Dr. Michael Johannes Pihl).

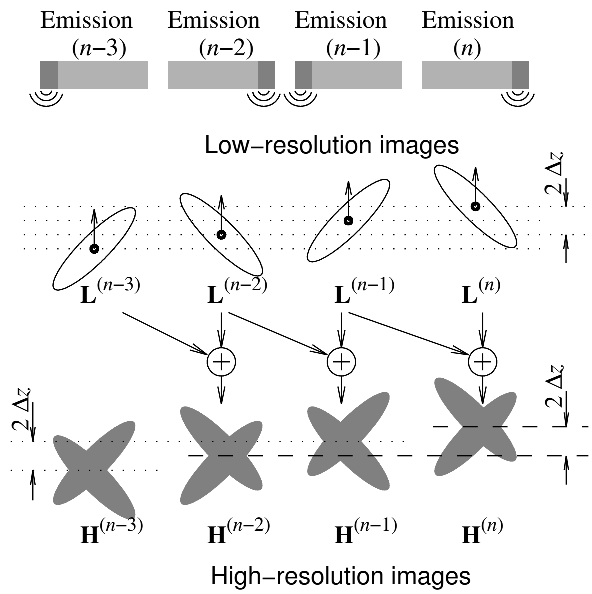

Figure 5.15: Acquisition of SA flow data and beamforming of high resolution images (from [93]).

flow imaging, which often has to resort to ECG gating when acquiring full volumes. Another drawback of traditional imaging is the use of transmit focusing. This cannot be made dynamic, and the images are only optimally focused at one single depth.

These problems can be solved by employing new imaging schemes based on synthetic aperture (SA) imaging [83, 84, 4, 85, 86, 87, 88, 89] and plane wave imaging [90, 91, 92]. Both these techniques insonify the whole region to interrogate and reconstruct the images during receive beamforming. This potentially can lead to very fast imaging and can also be used for flow imaging with very significant advantages.

The SA method is shown in Fig. 5.15. The transducer on the top emits a spherical wave, and the scattered signal is then received on all elements of the transducer. This process is repeated for a number of emission sources N_l on the aperture, and the data are collected for all receiving elements. From the received data for a single emission, a full low resolution (LR) image can be made. It is only focused in receive, but combining all the LR images yields a high resolution (HR) image. This is also focused during transmit as all the emitted fields are summed in phase [94]. The approach gives better focused images than traditional beamforming [95] with at least a preserved penetration depth when coded excitation is used.

The imaging scheme can also be used for flow estimation, although the data are acquired over a number of emissions, and therefore are shifted relative to each other. This is also illustrated in Fig. 5.15 for a short sequence. The point spread function for the low resolution images are shown below for a point scatterer moving towards the transducer. The LR point spread functions (PSFs) are different for the different emissions and can therefore not be directly correlated to find the velocity. Adding the low resolution images gives the PSF for the HR images, and it can be seen that these have the same shape, when the emission sequence combined is the same apart from the motion in position. The basic idea is therefore only to correlate the HR PSFs with the same emission sequence for finding the flow. This can also be performed recursively, so that a new correlation function is made for every new LR image [89].

The approach is illustrated in Fig. 5.16. The HR signals in one direction is shown on the top divided into segments. The length of the emission sequence is N_l and therefore emission n and $n + N_l$ can be correlated. This can then be averaged with $n+1$ correlated with $n+1+N_l$ as the time shift t_s is the same. It is therefore possible to continuously average the correlation function, and therefore use all data to get a very precise estimate of the correlation and thereby the velocity. This can be performed for all directions in the HR image continuously.

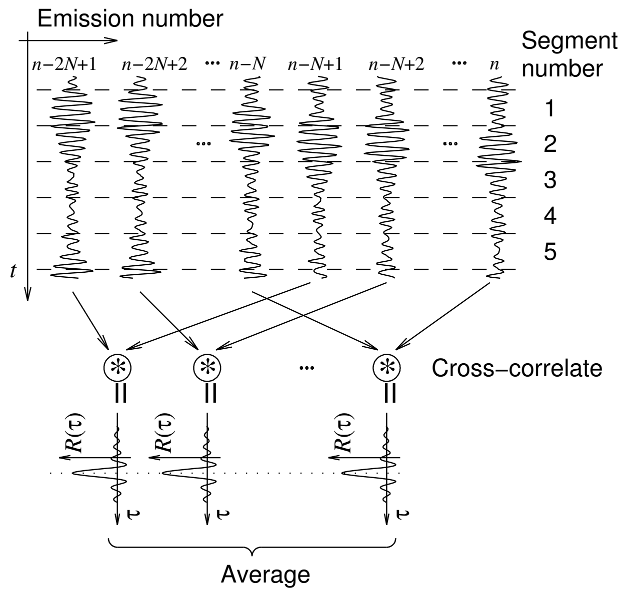

Figure 5.16: Averaging of cross-correlation functions in SA flow imaging (from [93]).

This has a number of advantages. The data can be acquired continuously, and data for flow imaging is, therefore, continuously available everywhere in the image. This makes it possible to average over very large amounts of data and makes echo canceling much easier [93]. Initialization effects for the filter can be neglected as the data are continuous, and this makes a large difference for *e.g.* low velocity flow. The cut-off frequency of the traditional echo canceling filter is proportional to f_{prf}/M where M here can be made arbitrarily large. The correlation estimates can also be averaged over a larger time interval T_i. The length N_h is only limited by the acceleration a_f of the flow. For a cross-correlation system there should at most be $\frac{1}{2}$ sampling interval shift due to acceleration:

$$a_f N_l T_{prf} N_h < \frac{1}{2 f_s T_{prf}} \frac{c}{2} \tag{5.49}$$

or

$$T_i = N_l T_{prf} N_h < \frac{f_{prf}}{2 f_s} \frac{c}{2 a_f} \tag{5.50}$$

to avoid de-correlation in the estimate of the cross-correlation function.

The data can also be focused in any direction as complete data sets are acquired, and the position of both the emitting sources and the receivers are known. The signals for velocity estimation can therefore be focused along the flow lines, if the beam-to-flow angle is known. This focusing scheme is shown in Fig.5.17. For each depth the data are focused along the flow and then used in a cross-correlation scheme to find the velocity [96, 97].

The estimated profiles for such a scheme are shown in Fig. 5.18 at a beam-to-flow angle of 60°. A linear array was used with an 8 emission SA sequence using a chirp pulse. Data from 64 elements were acquired for each emission, and 16 sequences, for a total of 128 emission, were averaged. All the 20 estimated velocity profiles are shown on the top and the mean ± 3 std are shown on the bottom. The mean relative standard deviation was 0.36% [96]. The approach also works for fully transverse flow and can yield a fast and quantitative display of the vector velocity.

It is also possible to determine the angle from the data. Here the directional lines are beamformed in all directions, and the one with the highest relative correlation indicates the angle [98]. An example of an in-vivo SA vector flow image from the carotid artery is shown in Fig. 5.19, where both velocities and angles have been estimated.

The data can also be used for visualizing the location of flow. This is done by finding the energy of the signals after echo canceling in a B-flow system [99] or power Doppler mode and show this. The intensity of the signal is then

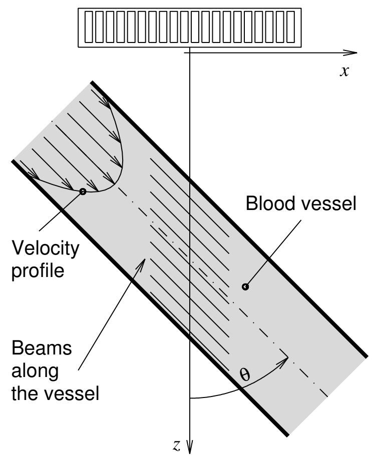

Figure 5.17: Directional beamforming along the flow lines (from [96]).

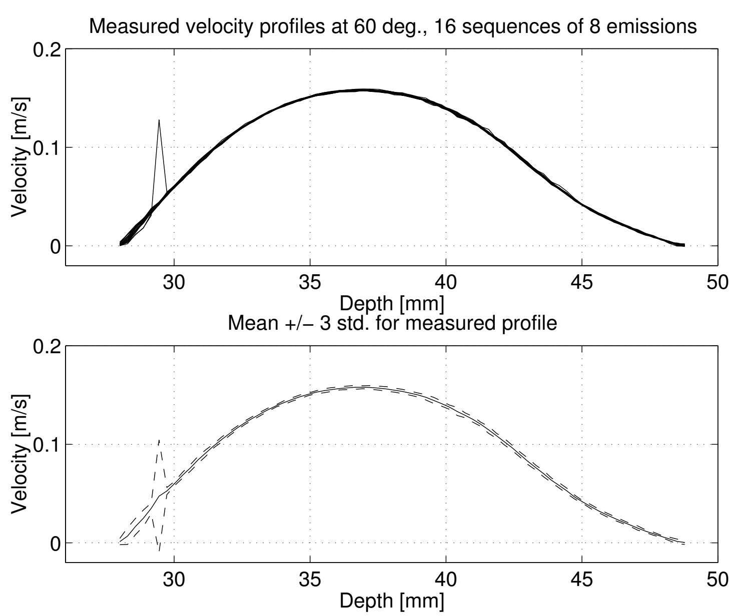

Figure 5.18: Estimated velocity profiles (top) at a beam-to-flow angle of 60°and mean value ± 3 standard deviations for SA vector flow imaging (from [96]).

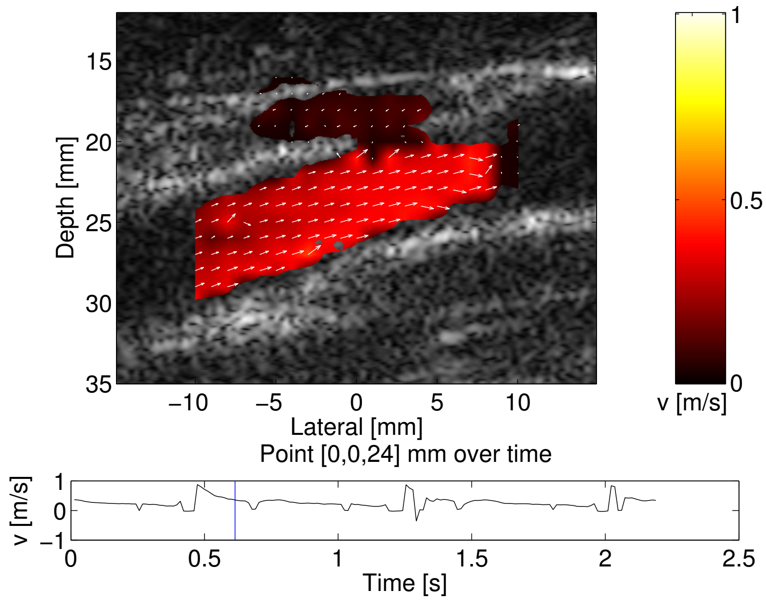

Figure 5.19: In-vivo SA vector flow imaging from the carotid artery (from [98])

roughly proportional to the velocity. An example of a SA B-flow image is shown in Fig. 5.20 at two different time instances in the cardiac cycle.

Another method for making fast and continuous imaging is to utilize plane wave emission. Here the full transducer is employed to transmit a plane wave, and then data are acquired for all the receiving elements [90]. The full image can then be reconstructed as for SA imaging. The image is only focused during receive and will have a lower resolution and higher side-lobes than conventional images. This can be compensated by using a number of plane waves at different angles as illustrated in Fig. 5.21. Combining these with a proper apodization can then lead to a full HR image [101]. This imaging scheme has the same advantages as SA imaging with a continuous data stream that can be used for increasing the sensitivity of flow estimation.

These imaging schemes can also be made very fast, and this is beneficial for looking at transitory and very fast flow phenomena, which are abundant in the human circulation [102, 103]. An plane wave vector flow image is shown in Fig. 5.22. A single plane wave was continuously emitted and the full image was beamformed for each emission. This was used in a speckle tracking scheme to find the velocity vectors and resulted in 100 independent vector velocity images per second [104]. A valve in the jugular vein and the carotid artery were imaged. The left image shows the open valve on the top, where a clockwise vortex is found behind the valve leaflets. The valve is incompetent and does not close correctly as shown in the right graph, where a noticeable reverse flow is seen. The vortex behind the leaflet has also changed direction. The middle image shows secondary rotational flow in the carotid artery during peak systole indicating the importance of having a full three-dimensional flow system.

SA and plane wave flow imaging is excellent for observing slow moving flow due to the long observation time possible. This has been demonstrated in [105, 101], which used plane wave imaging for mapping the brain function of a rat. The new acquisition methods can, thus, obtain data suitable for both fast vector velocity imaging and slow flow estimation for functional ultrasound imaging.

5.12 Motion estimation and other uses

The methods described can in general also be used for motion estimation. Tissue motion can be found by leaving out the echo canceling filter, and then all the methods can be applied for strain imaging [106], radiation force imaging

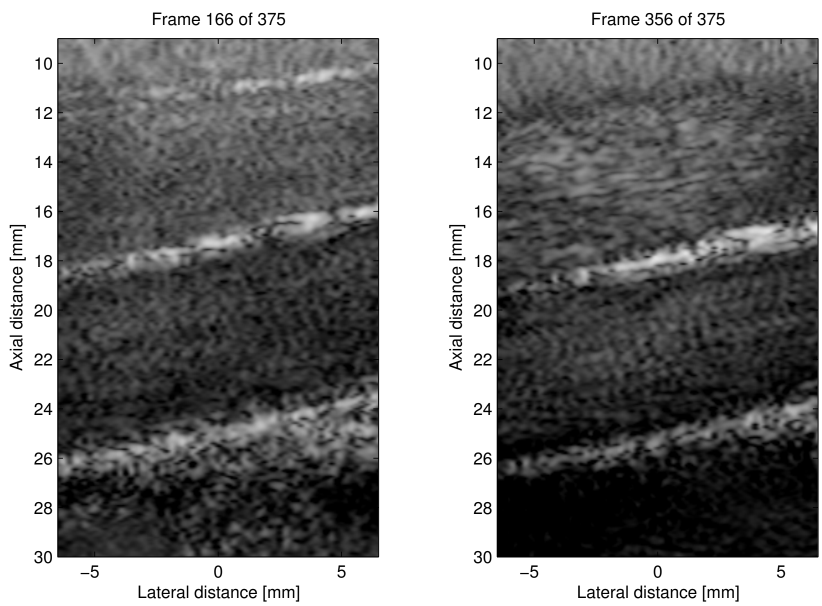

Figure 5.20: SA B-flow image of jugular vein and carotid artery (from [100]).

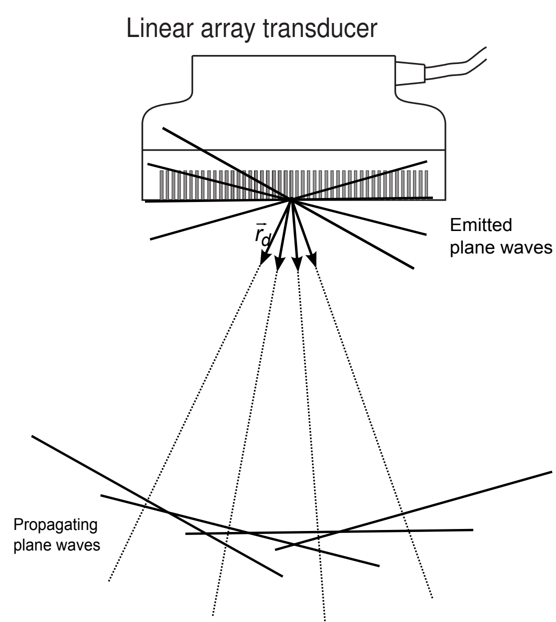

Figure 5.21: Plane wave imaging.

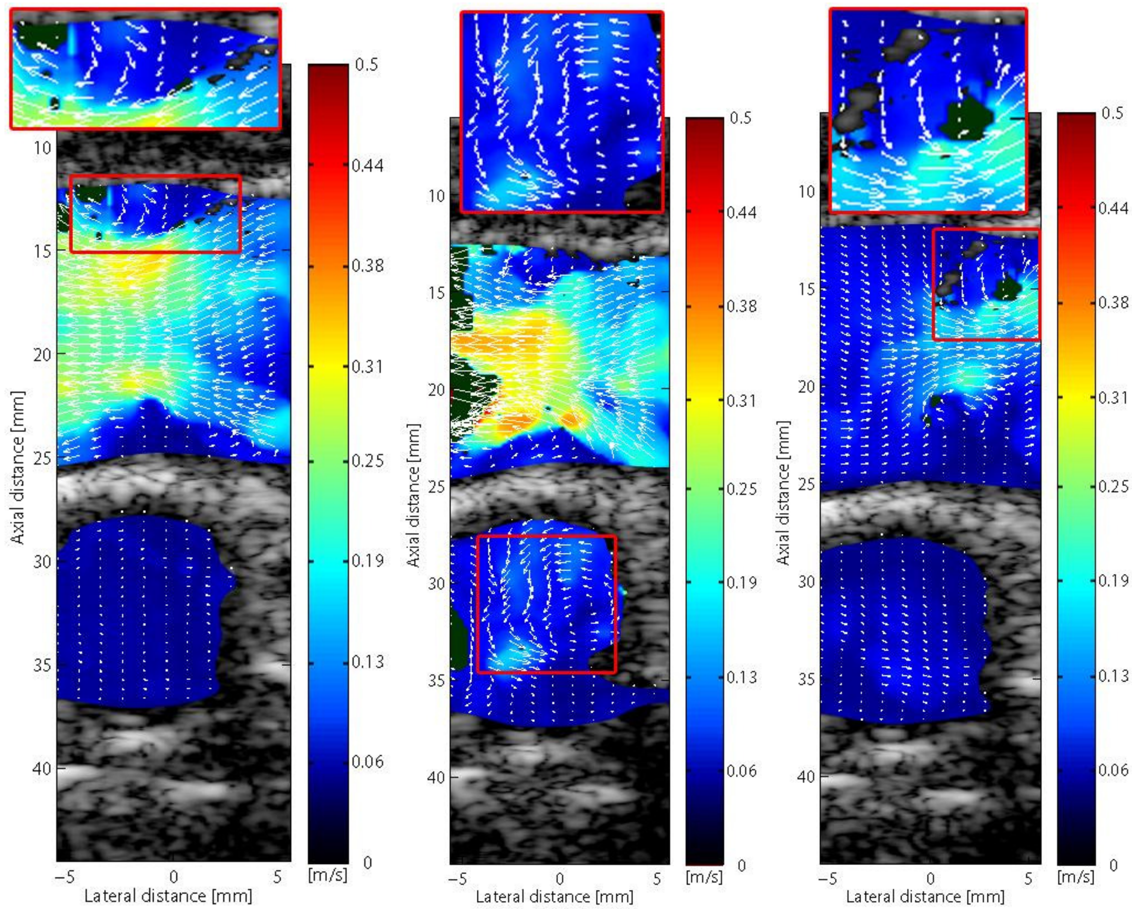

Figure 5.22: In-vivo plane wave vector flow images acquired at a frame rate of 100 Hz. The left image shows flow through a valve in the jugular vein at peak systole. The right image shows reverse flow during diastole. Note how the vortices behind the valve leaflet change rotation direction. Secondary flow is also seen in the middle image in the carotid artery below the jugular vein. (from [104]).

[107, 108], shear wave imaging [90], tissue Doppler [109] and others methods relying on the detection of motion or velocity. In general the methods have an improved performance for tissue, due to the increased signal-to-noise ratio and the lack of the echo canceling filter. It is therefore possible to calculate the derivatives necessary for some of these methods.

The velocity estimates are also used in deriving quantitative numbers useful for diagnostic purposes. Especially the new vector velocity estimates can be used for making the diagnosis more quantitative by calculating *e.g.* the volume flow [110], deriving quantities for indicating turbulence [111], and finding mean or peak velocities. It is also possible to use the vector velocity data for calculating flow gradients by solving the Navier-Stokes equations [112].

The development within velocity estimation is by no means complete. The combination of SA and plane wave imaging with 2-D and 3-D vector velocity and functional imaging is still a very active research area, and more complete information about the complex flow in the human body can be obtained. It will in real time reveal the many places for transient turbulences, vortices, and other multi-directional flow, and it will become possible to derive many more quantitative parameters for characterizing the patient's circulation.

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